Algebraic Geometry And Arithmetic Curves By Qing Liu

This book is about modern algebraic geometry. The title A Royal Road to Algebraic Geometry is inspired by the famous anecdote about the king asking Euclid if there really existed no simpler way for learning geometry, than to read all of his work Elements. Euclid is said to have answered: “There is no royal road to geometry!” The book starts by explaining this enigmatic answer, the aim of the book being to argue that indeed, in some sense there is a royal road to algebraic geometry. From a point of departure in algebraic curves, the exposition moves on to the present shape of the field, culminating with Alexander Grothendieck’s theory of schemes. Contemporary homological tools are explained. The reader will follow a directed path leading up to the main elements of modern algebraic geometry. When the road is completed, the reader is empowered to start navigating in this immense field, and to open up the door to a wonderful field of research. The greatest scientific experience of a lifetime!

This volume contains a collection of papers on algebraic curves and their applications. While algebraic curves traditionally have provided a path toward modern algebraic geometry, they also provide many applications in number theory, computer security and cryptography, coding theory, differential equations, and more. Papers cover topics such as the rational torsion points of elliptic curves, arithmetic statistics in the moduli space of curves, combinatorial descriptions of semistable hyperelliptic curves over local fields, heights on weighted projective spaces, automorphism groups of curves, hyperelliptic curves, dessins d'enfants, applications to Painlevé equations, descent on real algebraic varieties, quadratic residue codes based on hyperelliptic curves, and Abelian varieties and cryptography. This book will be a valuable resource for people interested in algebraic curves and their connections to other branches of mathematics.

This volume is the result of a (mainly) instructional conference on arithmetic geometry, held from July 30 through August 10, 1984 at the University of Connecticut in Storrs. This volume contains expanded versions of almost all the instructional lectures given during the conference. In addition to these expository lectures, this volume contains a translation into English of Falt ings' seminal paper which provided the inspiration for the conference. We thank Professor Faltings for his permission to publish the translation and Edward Shipz who did the translation. We thank all the people who spoke at the Storrs conference, both for helping to make it a successful meeting and enabling us to publish this volume. We would especially like to thank David Rohrlich, who delivered the lectures on height functions (Chapter VI) when the second editor was unavoidably detained. In addition to the editors, Michael Artin and John Tate served on the organizing committee for the conference and much of the success of the conference was due to them-our thanks go to them for their assistance. Finally, the conference was only made possible through generous grants from the Vaughn Foundation and the National Science Foundation.

The aim of these notes is to develop the theory of algebraic curves from the viewpoint of modern algebraic geometry, but without excessive prerequisites. We have assumed that the reader is familiar with some basic properties of rings, ideals and polynomials, such as is often covered in a one-semester course in modern algebra; additional commutative algebra is developed in later sections.

Complex Algebraic Curves

Algebraic Geometry

Basic Algebraic Geometry 2

Rational Points on Elliptic Curves

Part I: Schemes. With Examples and Exercises

This second volume introduces the concept of shemes, reviews some commutative algebra and introduces projective schemes. The finiteness theorem for coherent sheaves is proved, here again the techniques of homological algebra and sheaf cohomology are needed. In the last two chapters, projective curves over an arbitrary ground field are discussed, the theory of Jacobians is developed, and the existence of the Picard scheme is proved. Finally, the author gives some outlook into further developments- for instance é tale cohomology- and states some fundamental theorems.

Algebraic Geometry and Commutative Algebra in Honor of Masayoshi Nagata presents a collection of papers on algebraic geometry and commutative algebra in honor of Masayoshi Nagata for his significant contributions to commutative algebra. Topics covered range from power series rings and rings of invariants of finite linear groups to the convolution algebra of distributions on totally disconnected locally compact groups. The discussion begins with a description of several formulas for enumerating certain types of objects, which may be tabular arrangements of integers called Young tableaux or some types of monomials. The next chapter explains how to establish these enumerative formulas, with emphasis on the role played by transformations of determinantal polynomials and recurrence relations satisfied by them. The book then turns to several applications of the enumerative formulas and universal identity, including including enumerative proofs of the straightening law of Doubilet-Rota-Stein and computations of Hilbert functions of polynomial ideals of certain determinantal loci. Invariant differentials and quaternion extensions are also examined, along with the moduli of Todorov surfaces and the classification problem of embedded lines in characteristic p. This monograph will be a useful resource for practitioners and researchers in algebra and geometry.

Rapid, concise, self-contained introduction assumes only familiarity with elementary algebra. Subjects include algebraic varieties; products, projections, and correspondences; normal varieties; differential forms; theory of simple points; algebraic groups; more. 1958 edition.

This book introduces the reader to modern algebraic geometry. It presents Grothendieck's technically demanding language of schemes that is the basis of the most important developments in the last fifty years within this area. A systematic treatment and motivation of the theory is emphasized, using concrete examples to illustrate its usefulness. Several examples from the realm of Hilbert modular surfaces and of determinantal varieties are used methodically to discuss the covered techniques. Thus the reader experiences that the further development of the theory yields an ever better understanding of these fascinating objects. The text is complemented by many exercises that serve to check the comprehension of the text, treat further examples, or give an outlook on further results. The volume at hand is an introduction to schemes. To get startet, it requires only basic knowledge in abstract algebra and topology. Essential facts from commutative algebra are assembled in

an appendix. It will be complemented by a second volume on the cohomology of schemes.
Algebraic Curves and Riemann Surfaces
Schemes and Complex Manifolds
Arithmetic Geometry
Advanced Topics in the Arithmetic of Elliptic Curves
Function Theory, Geometry, Arithmetic

This book is a general introduction to the theory of schemes, followed by applications to arithmetic surfaces and to the theory of reduction of algebraic curves. The first part introduces basic objects such as schemes, morphisms, base change, local properties (normality, regularity, Zariski's Main Theorem). This is followed by the more global aspect: coherent sheaves and a finiteness theorem for their cohomology groups. Then follows a chapter on sheaves of differentials, dualizing sheaves, and Grothendieck's duality theory. The first part ends with the theorem of Riemann-Roch and its application to the study of smooth projective curves over a field. Singular curves are treated through a detailed study of the Picard group. The second part starts with blowing-ups and desingularisation (embedded or not) of fibered surfaces over a Dedekind ring that leads on to intersection theory on arithmetic surfaces. Castelnuovo's criterion is proved and also the existence of the minimal regular model. This leads to the study of reduction of algebraic curves. The case of elliptic curves is studied in detail. The book concludes with the funadmental theorem of stable reduction of Deligne-Mumford. The book is essentially self-contained, including the necessary material on commutative algebra. The prerequisites are therefore few, and the book should suit a graduate student. It contains many examples and nearly 600 exercises.

One of the great successes of twentieth century mathematics has been the remarkable qualitative understanding of rational and integral points on curves, gleaned in part through the theorems of Mordell, Weil, Siegel, and Faltings. It has become clear that the study of rational and integral points has deep connections to other branches of mathematics: complex algebraic geometry, Galois and étale cohomology, transcendence theory and diophantine approximation, harmonic analysis, automorphic forms, and analytic number theory. This text, which focuses on higher-dimensional varieties, provides precisely such an interdisciplinary view of the subject. It is a digest of research and survey papers by leading specialists; the book documents current knowledge in higher-dimensional arithmetic and gives indications for future research. It will be valuable not only to practitioners in the field, but to a wide audience of mathematicians and graduate students with an interest in arithmetic geometry. Contributors: Batyrev, V.V.; Broberg, N.; Colliot-Thélène, J-L.; Ellenberg, J.S.; Gille, P.; Graber, T.; Harari, D.; Harris, J.; Hassett, B.; Heath-Brown, R.; Mazur, B.; Peyre, E.; Poonen, B.; Popov, O.N.; Raskind, W.; Salberger, P.; Scharaschkin, V.; Shalika, J.; Starr, J.; Swinnerton-Dyer, P.; Takloo-Bighash, R.; Tschinkel, Y.: Voloch, J.F.; Wittenberg, O.

Geometry and the theory of numbers are as old as some of the oldest historical records of humanity. Ever since antiquity, mathematicians have discovered many beautiful interactions between the two subjects and recorded them in such classical texts as Euclid's Elements and Diophantus's Arithmetica. Nowadays, the field of mathematics that studies the interactions between number theory and algebraic geometry is known as arithmetic geometry. This book is an introduction to number theory and arithmetic geometry, and the goal of the text is to use geometry as the motivation to prove the main theorems in the book. For example, the fundamental theorem of arithmetic is a consequence of the tools we develop in order to find all the integral points on a line in the plane. Similarly, Gauss's law of quadratic reciprocity and the theory of continued fractions naturally arise when we attempt to determine the integral points on a curve in the plane given by a quadratic polynomial equation. After an introduction to the theory of diophantine equations, the rest of the book is structured in three acts that correspond to the study of the integral and rational solutions of linear, quadratic, and cubic curves, respectively. This book describes many applications including modern applications in cryptography; it also presents some recent results in arithmetic geometry. With many exercises, this book can be used as a text for a first course in number theory or for a subsequent course on arithmetic (or diophantine) geometry at the junior-senior level.

* Contains a selection of articles exploring geometric approaches to problems in algebra, algebraic geometry and number theory * The collection gives a representative sample of problems and most recent results in algebraic and arithmetic geometry * Text can serve as an intense introduction for graduate students and those wishing to pursue research in algebraic and arithmetic geometry

Basic Concepts, Coherent Cohomology, Curves and their Jacobians

Algebraic Curves in Cryptography

Barrett Lecture Series Conference, April 25-27, 2002, University of Tennessee, Knoxville, Tennessee
Arithmetic on Elliptic Curves with Complex Multiplication

The Geometry of Schemes

The book was easy to understand, with many examples. The exercises were well chosen, and served to give further examples and developments of the theory. --William Goldman, University of Maryland In this book, Miranda takes the approach that algebraic curves are best encountered for the first time over the complex numbers, where the reader's classical intuition about surfaces, integration, and other concepts can be brought into play. Therefore, many examples of algebraic curves are presented in the first chapters. In this way, the book begins as a primer on Riemann surfaces, with complex charts and meromorphic functions taking center stage. But the main examples come from projective curves, and slowly but surely the text moves toward the algebraic category. Proofs of the Riemann-Roch and Serre Duality Theorems are presented in an algebraic manner, via an adaptation of the adelic proof, expressed completely in terms of solving a Mittag-Leffler problem. Sheaves and cohomology are introduced as a unifying device in the latter chapters, so that their utility and naturalness are immediately obvious. Requiring a background of one semester of complex variable theory and a year of abstract algebra, this is an excellent graduate textbook for a second-semester course in complex variables or a year-long course in algebraic geometry.

This book is a general introduction to the theory of schemes, followed by applications to arithmetic surfaces and to the theory of reduction of algebraic curves. The first part introduces basic objects such as schemes, morphisms, base change, local properties (normality, regularity, Zariski's Main Theorem). This is followed by the more global aspect: coherent sheaves and a finiteness theorem for their cohomology groups. Then follows a chapter on sheaves of differentials, dualizing sheaves, and grothendieck's duality theory. The first part ends with the theorem of Riemann-Roch and its application to the study of smooth projective curves over a field. Singular curves are treated through a detailed study of the Picard group. The second part starts with blowing-ups and desingularization (embedded or not) of fibered surfaces over a Dedekind ring that leads on to intersection theory on arithmetic surfaces. Castelnuovo's criterion is proved and also the existence of the minimal regular model. This leads to the study of reduction of algebraic curves. The case of elliptic curves is studied in detail. The book concludes with the fundamental theorem of stable reduction of Deligne-Mumford. The book is essentially self-contained, including the necessary material on commutative algebra. The prerequisites are therefore few, and the book should suit a graduate student. It contains many examples and nearly 600 exercises.

An introductory 1997 account in the style of the original discoverers, treating the fundamental themes even-handedly.

The theory of elliptic curves involves a blend of algebra, geometry, analysis, and number theory. This book stresses this interplay as it develops the basic theory, providing an opportunity for readers to appreciate the unity of modern mathematics. The book’s accessibility, the informal writing style, and a wealth of exercises make it an ideal introduction for those interested in learning about Diophantine equations and arithmetic geometry.

Algebra, Arithmetic, and Geometry

3264 and All That

Arithmetic and Geometry of K3 Surfaces and Calabi–Yau Threefolds

An Invitation to Arithmetic Geometry

Arithmetic of Algebraic Curves

3264, the mathematical solution to a question concerning geometric figures.

Algebraic geometry is a fascinating branch of mathematics that combines methods from both, algebra and geometry. It transcends the limited scope of pure algebra by means of geometric construction principles. Moreover, Grothendieck’s schemes invented in the late 1950s allowed the application of algebraic-geometric methods in fields that formerly seemed to be far away from geometry, like algebraic number theory. The new techniques paved the way to spectacular progress such as the proof of Fermat’s Last Theorem by Wiles and Taylor. The scheme-theoretic approach to algebraic geometry is explained for non-experts. More advanced readers can use the book to broaden their view on the subject. A separate part deals with the necessary prerequisites from commutative algebra. On a whole, the book provides a very accessible and self-contained introduction to algebraic geometry, up to a quite advanced level. Every chapter of the book is preceded by a motivating introduction with an informal discussion of the contents. Typical examples and an abundance of exercises illustrate each section. This way the book is an excellent solution for learning by yourself or for complementing knowledge that is already present. It can equally be used as a convenient source for courses and seminars or as supplemental literature.

Grothendieck’s beautiful theory of schemes permeates modern algebraic geometry and underlies its applications to number theory, physics, and applied mathematics. This simple account of that theory emphasizes and explains the universal geometric concepts behind the definitions. In the book, concepts are illustrated with fundamental examples, and explicit calculations show how the constructions of scheme theory are carried out in practice.

EMAlgebra, Arithmetic, and Geometry: In Honor of Yu. I. ManinEM consists of invited expository and research articles on new developments arising from Manin’s outstanding contributions to mathematics.

Algebraic Geometry and Commutative Algebra

With Examples and Exercises

A Royal Road to Algebraic Geometry

An Introduction to Algebraic Geometry

In Honor of Masayoshi Nagata

Conference proceedings based on the 1996 LMS Durham Symposium 'Galois representations in arithmetic algebraic geometry'.

Capacity is a measure of size for sets, with diverse applications in potential theory, probability and number theory. This book lays foundations for a theory of capacity for adelic sets on algebraic curves. Its main result is an arithmetic one, a generalization of a theorem of Fekete and Szegö which gives a sharp existence/finiteness criterion for algebraic points whose conjugates lie near a specified set on a curve. The book brings out a deep connection between the classical Green's functions of analysis and Néron's local height pairings; it also points to an interpretation of capacity as a kind of intersection index in the framework of Arakelov

Theory. It is a research monograph and will primarily be of interest to number theorists and algebraic geometers; because of applications of the theory, it may also be of interest to logicians. The theory presented generalizes one due to David Cantor for the projective line. As with most adelic theories, it has a local and a global part. Let /K be a smooth, complete curve over a global field; let Kv denote the algebraic closure of any completion of K. The book first develops capacity theory over local fields, defining analogues of the classical logarithmic capacity and Green's functions for sets in (Kv). It then develops a global theory, defining the capacity of a galois-stable set in (Kv) relative to an effictive global algebraic divisor. The main technical result is the construction of global algebraic functions whose logarithms closely approximate Green's functions at all places of K. These functions are used in proving the generalized Fekete-Szegö theorem; because of their mapping properties, they may be expected to have other applications as well.

Arithmetic algebraic geometry is in a fascinating stage of growth, providing a rich variety of applications of new tools to both old and new problems. Representative of these recent developments is the notion of Arakelov geometry, a way of "completing" a variety over the ring of integers of a number field by adding fibres over the Archimedean places. Another is the appearance of the relations between arithmetic geometry and Nevanlinna theory, or more precisely between diophantine approximation theory and the value distribution theory of holomorphic maps. Research mathematicians and graduate students in algebraic geometry and number theory will find a valuable and lively view of the field in this state-of-the-art selection.

Aimed primarily at graduate students and beginning researchers, this book provides an introduction to algebraic geometry that is particularly suitable for those with no previous contact with the subject; it assumes only the standard background of undergraduate algebra. The book starts with easily-formulated problems with non-trivial solutions and uses these problems to introduce the fundamental tools of modern algebraic geometry: dimension; singularities; sheaves; varieties; and cohomology. A range of exercises is provided for each topic discussed, and a selection of problems and exam papers are collected in an appendix to provide material for further study.

Algebraic Geometry and Arithmetic Curves

Algebraic Curves

Volume I: In Honor of Yu. I. Manin

An Introduction

Arithmetic Algebraic Geometry

This proceedings volume resulted from the John H. Barrett Memorial Lecture Series held at the University of Tennessee (Knoxville). The articles reflect recent developments in algebraic geometry. It is suitable for graduate students and researchers interested in algebra and algebraic geometry.

The reach of algebraic curves in cryptography goes far beyond elliptic curve or public key cryptography yet these other application areas have not been systematically covered in the literature. Addressing this gap, Algebraic Curves in Cryptography explores the rich uses of algebraic curves in a range of cryptographic applications, such as secret sh

This development of the theory of complex algebraic curves was one of the peaks of nineteenth century mathematics. They have many fascinating properties and arise in various areas of mathematics, from number theory to theoretical physics, and are the subject of much research. By using only the basic techniques acquired in most undergraduate courses in mathematics, Dr. Kirwan introduces the theory, observes the algebraic and topological properties of complex algebraic curves, and shows how they are related to complex analysis.

Author S.A. Stepanov thoroughly investigates the current state of the theory of Diophantine equations and its related methods. Discussions focus on arithmetic, algebraic-geometric, and logical aspects of the problem. Designed for students as well as researchers, the book includes over 250 excercises accompanied by hints, instructions, and references. Written in a clear manner, this text does not require readers to have special knowledge of modern methods of algebraic geometry.

Algebraic Curves and Their Applications

Lectures on Algebraic Geometry II

Elliptic Curves

Arithmetic of Higher-Dimensional Algebraic Varieties

The Arithmetic of Elliptic Curves

The second volume of Shafarevich's introductory book on algebraic geometry focuses on schemes, complex algebraic varieties and complex manifolds. As with first volume the author has revised the text and added new material. Although the material is more advanced than in Volume 1 the algebraic apparatus is kept to a minimum making the book accessible to non-specialists. It can be read independently of the first volume and is suitable for beginning graduate students.

This book features recent developments in a rapidly growing area at the interface of higher-dimensional birational geometry and arithmetic geometry. It focuses on the geometry of spaces of rational curves, with an emphasis on applications to arithmetic questions. Classically, arithmetic is the study of rational or integral solutions of diophantine equations and geometry is the study of lines and conics. From the modern standpoint, arithmetic is the study of rational and integral points on algebraic varieties over nonclosed fields. A major insight of the 20th century was that arithmetic properties of an algebraic variety are tightly linked to the geometry of rational curves on the variety and how they vary in families. This collection of solicited survey and research papers is intended to serve as an introduction for graduate students and researchers interested in entering the field, and as a source of reference for experts working on related problems. Topics that will be addressed include: birational properties such as rationality, unirationality, and rational connectedness, existence of rational curves in prescribed homology classes, cones of rational curves on rationally connected and Calabi-Yau varieties, as well as related questions within the framework of the Minimal Model Program.

In the introduction to the first volume of The Arithmetic of Elliptic Curves (Springer-Verlag, 1986), I observed that "the theory of elliptic curves is rich, varied, and amazingly vast," and as a consequence, "many important topics had to be omitted." I included a brief introduction to ten additional topics as an appendix to the first volume, with the tacit understanding that eventually there might be a second volume containing the details. You are now holding that second volume. it turned out that even those ten topics would not fit Unfortunately, into a single book, so I was forced to make some choices. The following material is covered in this book: I. Elliptic and modular functions for the full modular group. II. Elliptic curves with complex multiplication. III. Elliptic surfaces and specialization theorems. IV. Neron models, Kodaira-Neron classification of special fibers, Tate's algorithm, and Ogg's conductor-discriminant formula. V. Tate's theory of q-curves over p-adic fields. VI. Neron's theory of canonical local height functions.

An introduction to abstract algebraic geometry, with the only prerequisites being results from commutative algebra, which are stated as needed, and some elementary topology. More than 400 exercises distributed throughout the book offer specific examples as well as more specialised topics not treated in the main text, while three appendices present brief accounts of some areas of current research. This book can thus be used as textbook for an introductory course in algebraic geometry following a basic graduate course in algebra. Robin Hartshorne studied algebraic geometry with Oscar Zariski and David Mumford at Harvard, and with J.-P. Serre and A. Grothendieck in Paris. He is the author of "Residues and Duality", "Foundations of Projective Geometry", "Ample Subvarieties of Algebraic Varieties", and numerous research titles.

Introduction to Algebraic Geometry

Geometric Methods in Algebra and Number Theory

Capacity Theory on Algebraic Curves

Recent Progress in Arithmetic and Algebraic Geometry

A Second Course in Algebraic Geometry

Extremely carefully written, masterfully thought out, and skillfully arranged introduction ... to the arithmetic of algebraic curves, on the one hand, and to the algebro-geometric aspects of number theory, on the other hand. ... an excellent guide for beginners in arithmetic geometry, just as an interesting reference and methodical inspiration for teachers of the subject ... a highly welcome addition to the existing literature. --Zentralblatt MATH The interaction between number theory and algebraic geometry has been especially fruitful. In this volume, the author gives a unified presentation of some of the basic tools and concepts in number theory, commutative algebra, and algebraic geometry, and for the first time in a book at this level, brings out the deep analogies between them. The geometric viewpoint is stressed throughout the book. Extensive examples are given to illustrate each new concept, and many interesting exercises are given at the end of each chapter. Most of the important results in the one-dimensional case are proved, including Bombieri's proof of the Riemann Hypothesis for curves over a finite field. While the book is not intended to be an introduction to schemes, the author indicates how many of the geometric notions introduced in the book relate to schemes, which will aid the reader who goes to the next level of this rich subject.

This new-in-paperback edition provides a general introduction to algebraic and arithmetic geometry, starting with the theory of schemes, followed by applications to arithmetic surfaces and to the theory of reduction of algebraic curves. The first part introduces basic objects such as schemes, morphisms, base change, local properties (normality, regularity, Zariski's Main Theorem). This is followed by the more global aspect: coherent sheaves and a finiteness theorem for their cohomology groups. Then follows a chapter on sheaves of differentials, dualizing sheaves, and Grothendieck's duality theory. The first part ends with the theorem of Riemann-Roch and its application to the study of smooth projective curves over a field. Singular curves are treated through a detailed study of the Picard group. The second part starts with blowing-ups and desingularisation (embedded or not) of fibered surfaces over a Dedekind ring that leads on to intersection theory on arithmetic surfaces. Castelnuovo's criterion is proved and also the existence of the minimal regular model. This leads to the study of reduction of algebraic curves. The case of elliptic curves is studied in detail. The book concludes with the fundamental theorem of stable reduction of Deligne-Mumford. This book is essentially self-contained, including the necessary material on commutative algebra. The prerequisites are few, and including many examples and approximately 600 exercises, the book is ideal for graduate students.

The first application of modern algebraic techniques to a comprehensive selection of classical geometric problems. Written with spirit and originality, this is a valuable book for anyone interested in the subject from other than the purely algebraic point of view. Originally published in 1953. The Princeton Legacy Library uses the latest print-on-demand technology to again make available previously out-of-print books from the distinguished backlist of Princeton University Press. These editions preserve the original texts of these important books while presenting them in durable paperback and hardcover editions. The goal of the Princeton Legacy Library is to vastly increase access to the rich scholarly heritage found in the thousands of books published by Princeton University Press since its founding in 1905.

In recent years, research in K3 surfaces and Calabi–Yau varieties has seen spectacular progress from both arithmetic and geometric points of view, which in turn continues to have a huge influence and impact in theoretical physics—in particular, in string theory. The workshop on Arithmetic and Geometry of K3 surfaces and Calabi–Yau threefolds, held at the Fields Institute (August 16-25, 2011), aimed to give a state-of-the-art survey of these new developments. This proceedings volume includes a representative sampling of the broad range of topics covered by the workshop. While the subjects range from arithmetic geometry through algebraic geometry and differential geometry to mathematical physics, the papers are naturally related by the common theme of Calabi–Yau varieties. With the big variety of branches of mathematics and mathematical physics touched upon, this area reveals many deep connections between subjects previously considered unrelated. Unlike most other conferences, the 2011 Calabi–Yau workshop started with 3 days of introductory lectures. A selection of 4 of these lectures is included in this volume. These lectures can be used as a starting point for the graduate students and other junior researchers, or as a guide to the subject.

Galois Representations in Arithmetic Algebraic Geometry
Number Theory and Geometry: An Introduction to Arithmetic Geometry
Algebraic Geometry I: Schemes
Birational Geometry, Rational Curves, and Arithmetic

The theory of elliptic curves is distinguished by its long history and by the diversity of the methods that have been used in its study. This book treats the arithmetic approach in its modern formulation, through the use of basic algebraic number theory and algebraic geometry. Following a brief discussion of the necessary algebro-geometric results, the book proceeds with an exposition of the geometry and the formal group of elliptic curves, elliptic curves over finite fields, the complex numbers, local fields, and global fields. Final chapters deal with integral and rational points, including Siegels theorem and explicit computations for the curve Y = X + DX, while three appendices conclude the whole: Elliptic Curves in Characteristics 2 and 3, Group Cohomology, and an overview of more advanced topics.