Algebraic Geometry And Statistical Learning Theory

This book presents algorithmic tools for algebraic geometry, with experimental applications. It also introduces Macaulay 2, a computer algebra system supporting research in algebraic geometry, commutative algebra, and their applications. The algorithmic tools presented here are designed to serve readers wishing to bring such tools to bear on their own problems. The first part of the book covers Macaulay 2 using concrete applications; the second emphasizes details of the mathematics.

The book developed from the need to teach a linear algebra course to students focused on data science and bioinformatics programs. These students tend not to realize the importance of linear algebra in applied sciences since traditional linear algebra courses tend to cover mathematical contexts but not the computational aspect of linear algebra or its applications to data science and bioinformatics. The author presents the topics in a traditional course yet offers lectures as well as lab exercises on simulated and empirical data sets. This textbook provides students a theoretical basis which can then be applied to the practical R and Python problems, providing the tools needed for real-world applications. Each section starts with working examples to demonstrate how tools from linear algebra can help solve problems in applied science. These exercises start from easy computations, such as computing determinants of matrices, to practical applications on simulated and empirical data sets with R so that students learn how to get started with R along with computational examples in each section and then they learn how to apply what they learn to problems in applied sciences. This book is designed from first principles to demonstrate the importance of linear algebra through working computational examples with R and python including tutorials on how to install R in the Appendix. If a student has never seen R, they can get started without any additional help. Since Python is one of the most popular languages in data science, optimization, and computer science, code supplements are available for students who feel more comfortable with Python. R is used primarily for computational examples to develop student's practical computational skills. Table of Contents Preface List of Figures List of Tables 1. Systems of Linear Equations and Matrices 2. Matrix Arithmetic 3. Deteminants 4. Vector Spaces 5. Inner Product Space 6. Eigen values and Eigen vectors 7. Linear Regression 8. Linear Programming Network Analysis Appendices A) Introduction to RStudio via Amazon Web Service (AWS) B) Introduction to R Bibliography Index Biography Dr. Ruriko Yoshida is an Associate Professor of Operations Research at the Naval Postgraduate School. She received her Ph.D. in Mathematics from the University of California, Davis. Her research topics cover a wide variety of areas: applications of algebraic combinatorics to statistical problems such as statistical learning on non-Euclidean spaces, sensor networks, phylogenetics, and phylogenomics. She teaches courses in statistics, stochastic models, probability, and data science.

This volume contains the proceedings of the AMS Special Session on Algebraic and Geometric Methods in Applied

Discrete Mathematics, held on January 11, 2015, in San Antonio, Texas. The papers present connections between techniques from “pure” mathematics and various applications amenable to the analysis of discrete models, encompassing applications of combinatorics, topology, algebra, geometry, optimization, and representation theory. Papers not only present novel results, but also survey the current state of knowledge of important topics in applied discrete mathematics. Particular highlights include: a new computational framework, based on geometric combinatorics, for structure prediction from RNA sequences; a new method for approximating the optimal solution of a sum of squares problem; a survey of recent Helly-type geometric theorems; applications of representation theory to voting theory and game theory; a study of fixed points of tensors; and exponential random graph models from the perspective of algebraic statistics with applications to networks. This volume was written for those trained in areas such as algebra, topology, geometry, and combinatorics who are interested in tackling problems in fields such as biology, the social sciences, data analysis, and optimization. It may be useful not only for experts, but also for students who wish to gain an applied or interdisciplinary perspective.

How does an algebraic geometer studying secant varieties further the understanding of hypothesis tests in statistics? Why would a statistician working on factor analysis raise open problems about determinantal varieties? Connections of this type are at the heart of the new field of "algebraic statistics". In this field, mathematicians and statisticians come together to solve statistical inference problems using concepts from algebraic geometry as well as related computational and combinatorial techniques. The goal of these lectures is to introduce newcomers from the different camps to algebraic statistics. The introduction will be centered around the following three observations: many important statistical models correspond to algebraic or semi-algebraic sets of parameters; the geometry of these parameter spaces determines the behaviour of widely used statistical inference procedures; computational algebraic geometry can be used to study parameter spaces and other features of statistical models.

Algebraic Geometry and Statistical Learning Theory

An Introduction to Algebraic Geometry

Fundamentals of Mathematical Statistics

Computational Algebraic Geometry

Sharpening Mathematical Analysis Skills

Drawing from philosophical work on the nature of concepts and from empirical studies of visual perception, mental imagery, and numerical cognition, Giaquinto explores a major source of our grasp of mathematics, using examples from basic geometry, arithmetic, algebra, and real analysis.

Algebraic Geometry and Statistical Learning TheoryCambridge University Press

Geometry and Statistics, Volume 46 in the Handbook of Statistics series, highlights new advances in the field, with this new volume presenting interesting chapters written by an international board of authors. Provides the authority and expertise of leading contributors from an international board of authors Presents the latest release in the Handbook of Statistics series Updated release includes the latest information on Geometry and Statistics

The theory and practice of computation in algebraic geometry and related domains, from a mathematical point of view, has generated an increasing interest both for its rich theoretical possibilities and its usefulness in applications in science and engineering. In fact, it is one of the master keys for future significant improvement of the computer algebra systems (e.g., Reduce, Macsyma, Maple, Mathematica, Axiom, Macaulay, etc.) that have become such useful tools for many scientists in a variety of disciplines. The major themes covered in this volume, arising from papers p- sented at the conference MEGA-92 were: - Effective methods and complexity issues in commutative algebra, projective geometry, real geometry, and algebraic number theory - Algebra-geometric methods in algebraic computing and applica tions. MEGA-92 was the second of a new series of European conferences on the general theme of Effective Methods in Algebraic Geometry. It was held in Nice, France, on April 21-25, 1992 and built on the themes presented at MEGA-90 (Livomo, Italy, April 17-21, 1990). The next conference - MEGA-94 - will be held in Santander, Spain in the spring of 1994. The Organizing committee that initiatiod and supervises this bi enniel conference consists of A. Conte (Torino), J.H. Davenport (Bath), A. Galligo (Nice), D. Yu. Grigoriev (Petersburg), J. Heintz (Buenos Aires), W. Lassner (Leipzig), D. Lazard (paris), H.M. MOller (Hagen), T. Mora (Genova), M. Pohst (DUsseldort), T. Recio (Santander), J.J.

Combinatorial Algebraic Geometry

Geometry and Statistics

Semialgebraic Statistics and Latent Tree Models

An Invitation to Quantum Cohomology

Statistical Learning with Sparsity

Sure to be influential, this book lays the foundations for the use of algebraic geometry in statistical learning theory. Many widely used statistical models and learning machines applied to information science have a parameter space that is singular: mixture models, neural networks, HMMs, Bayesian networks, and stochastic context-free grammars are major examples. Algebraic geometry and singularity theory provide the necessary tools for studying such non-smooth models. Four main formulas are established: 1. the log likelihood function can be given a common standard form using resolution of singularities, even applied to more complex models; 2. the asymptotic behaviour of the marginal likelihood or 'the evidence' is

derived based on zeta function theory; 3. new methods are derived to estimate the generalization errors in Bayes and Gibbs estimations from training errors; 4. the generalization errors of maximum likelihood and a posteriori methods are clarified by empirical process theory on algebraic varieties.

Combinatorics and Algebraic Geometry have enjoyed a fruitful interplay since the nineteenth century. Classical interactions include invariant theory, theta functions and enumerative geometry. The aim of this volume is to introduce recent developments in combinatorial algebraic geometry and to approach algebraic geometry with a view towards applications, such as tensor calculus and algebraic statistics. A common theme is the study of algebraic varieties endowed with a rich combinatorial structure. Relevant techniques include polyhedral geometry, free resolutions, multilinear algebra, projective duality and compactifications.

This book gathers together a novel collection of problems in mathematical analysis that are challenging and worth studying. They cover most of the classical topics of a course in mathematical analysis, and include challenges presented with an increasing level of difficulty. Problems are designed to encourage creativity, and some of them were especially crafted to lead to open problems which might be of interest for students seeking motivation to get a start in research. The sets of problems are comprised in Part I. The exercises are arranged on topics, many of them being preceded by supporting theory. Content starts with limits, series of real numbers and power series, extending to derivatives and their applications, partial derivatives and implicit functions. Difficult problems have been structured in parts, helping the reader to find a solution. Challenges and open problems are scattered throughout the text, being an invitation to discover new original methods for proving known results and establishing new ones. The final two chapters offer ambitious readers splendid problems and two new proofs of a famous quadratic series involving harmonic numbers. In Part II, the reader will find solutions to the proposed exercises. Undergraduate students in mathematics, physics and engineering, seeking to strengthen their skills in analysis, will most benefit from this work, along with instructors involved in math contests, individuals who want to enrich and test their knowledge in analysis, and anyone willing to explore the standard topics of mathematical analysis in ways that aren’t commonly seen in regular textbooks.

This introduction to algebraic geometry allows readers to grasp the fundamentals of the subject with only linear algebra and calculus as prerequisites. After a brief history of the subject, the book introduces projective spaces and projective varieties, and explains plane curves and resolution of their singularities. The volume further develops the geometry of algebraic curves and treats congruence zeta functions of algebraic curves over a finite field. It concludes with a complex analytical discussion of algebraic curves. The author emphasizes computation of concrete examples rather than proofs, and these examples are discussed from various viewpoints. This approach allows readers to develop a deeper understanding of the theorems.

Solving Systems of Polynomial Equations
Differential Geometry in Statistical Inference
Algebraic and Geometric Methods in Statistics
Kontsevich's Formula for Rational Plane Curves
The Lasso and Generalizations

This book is a true introduction to the basic concepts and techniques of algebraic geometry. The language is purposefully kept on an elementary level, avoiding sheaf theory and cohomology theory. The introduction of new algebraic concepts is always motivated by a discussion of the corresponding geometric ideas. The main point of the book is to illustrate the interplay between abstract theory and specific examples. The book contains numerous problems that illustrate the general theory. The text is suitable for advanced undergraduates and beginning graduate students. It contains sufficient material for a one-semester course. The reader should be familiar with the basic concepts of modern algebra. A course in one complex variable would helpful, but is not necessary.

The aim of this book is to present the mathematics underlying elementary statistical methods in as simple a manner as possible. These methods incluc independent and paired sample t-tests, analysis of variance, regression, and the analysis of covariance. The author's principle tool is the use of geome ideas to provide more visual insight and to make the theory accessible to a wider audience than is usually possible.

An introduction to abstract algebraic geometry, with the only prerequisites being results from commutative algebra, which are stated as needed, and so elementary topology. More than 400 exercises distributed throughout the book offer specific examples as well as more specialised topics not treated in main text, while three appendices present brief accounts of some areas of current research. This book can thus be used as textbook for an introductory course in algebraic geometry following a basic graduate course in algebra. Robin Hartshorne studied algebraic geometry with Oscar Zariski and David Mumford at Harvard, and with J.-P. Serre and A. Grothendieck in Paris. He is the author of "Residues and Duality", "Foundations of Projective Geometry", "Ample Subvarieties of Algebraic Varieties", and numerous research titles.

Several years ago our statistical friends and relations introduced us to the work of Amari and Barndorff-Nielsen on applications of differential geometry statistics. This book has arisen because we believe that there is a deep relationship between statistics and differential geometry and moreoever that th relationship uses parts of differential geometry, particularly its 'higher-order' aspects not readily accessible to a statistical audience from the existing literature. It is, in part, a long reply to the frequent requests we have had for references on differential geometry! While we have not gone beyond the p breaking work of Amari and Barndorff- Nielsen in the realm of applications, our book gives some new explanations of their ideas from a first principles point of view as far as geometry is concerned. In particular it seeks to explain why geometry should enter into parametric statistics, and how the theory asymptotic expansions involves a form of higher-order differential geometry. The first chapter of the book explores exponential families as flat geometr Indeed the whole notion of using log-likelihoods amounts to exploiting a particular form of flat space known as an affine geometry, in which straight line and planes make sense, but lengths and angles are absent. We use these geometric ideas to introduce the notion of the second fundamental form of a whose vanishing characterises precisely the exponential families.

Counting Surfaces

Algebraic Geometry

Introduction to Algebraic Geometry

Computations in Algebraic Geometry with Macaulay 2

Elementary Algebraic Geometry

Mathematical Theory of Bayesian Statistics introduces the mathematical foundation of Bayesian inference which is well-known to be more accurate in many real-world problems than the maximum likelihood method. Recent research has uncovered several mathematical laws in Bayesian statistics, by which both the generalization loss and the marginal likelihood are estimated even if the posterior distribution cannot be approximated by any normal distribution. Features Explains Bayesian inference not subjectively but objectively. Provides a mathematical framework for conventional Bayesian theorems. Introduces and proves new theorems. Cross validation and information criteria of Bayesian statistics are studied from the mathematical point of view. Illustrates applications to several statistical problems, for example, model selection, hyperparameter optimization, and hypothesis tests. This book provides basic introductions for students, researchers, and users of Bayesian statistics, as well as applied mathematicians. Author Sumio Watanabe is a professor of Department of Mathematical and Computing Science at Tokyo Institute of Technology. He studies the relationship between algebraic geometry and mathematical statistics.

Introduces machine learning and its algorithmic paradigms, explaining the principles behind automated learning approaches and the considerations underlying their usage.

The problem of enumerating maps (a map is a set of polygonal "countries" on a world of a certain topology, not necessarily the plane or the sphere) is an important problem in mathematics and physics, and it has many applications ranging from statistical physics, geometry, particle physics, informatics, biology, ... etc. This problem has been studied by many communities of researchers, mostly combinatorists, probabilists, and physicists. In 1978+, physicists have invented a method called "matrix models" to address that problem, and many results have been obtained. Besides, another important problem in mathematics and physics (in particular string theory), is to count Riemann surfaces. Riemann surfaces of a given topology are parametrized by a finite number of real parameters (called moduli), and the moduli space is a finite dimensional compact manifold of complicated topology. The number of Riemann surfaces is the volume of that moduli space. More generally, an important problem in algebraic geometry is to characterize the moduli spaces, by computing not only their volumes, but also their intersection numbers. The so called Witten's conjecture (which was first proved by Kontsevich), was the assertion that Riemann surfaces can be obtained as limits of polygonal surfaces (maps), made of a very large number of very small polygons. In other words, the number of maps in a certain limit, should give the intersection numbers of moduli spaces. In this book, we show how that limit takes place. The goal of this book is to explain the "matrix model" method, to show the main results obtained with it, and to compare it with methods used in combinatorics (bijective proofs, Tutte's equations), or algebraic geometry (Mirzakhani's recursions). The book intends to be self-contained and pedagogical, and will provide comprehensive proofs, several examples, and will give the general formula for the enumeration of maps on surfaces of any topology. In the end, the link with more general topics such as algebraic geometry, string theory, will be discussed, and in particular we give a proof of the Witten-Kontsevich conjecture.

Information geometry provides the mathematical sciences with a new framework of analysis. It has emerged from the investigation of the natural differential geometric structure on manifolds of probability distributions, which consists of a Riemannian metric defined by the Fisher information and a one-parameter family of affine connections called the α-connections. The duality between the α-connection and the $(-\alpha)$-connection together with the metric play an essential role in this geometry. This kind of duality, having emerged from manifolds of probability distributions, is ubiquitous, appearing in a variety of problems which might have no explicit relation to probability theory. Through the duality, it is possible to analyze various fundamental problems in a unified perspective. The first half of this book is devoted to a comprehensive introduction to the mathematical foundation of information geometry, including preliminaries from differential geometry, the geometry of manifolds or probability distributions, and the general theory of dual affine connections. The second half of the text provides an overview of many areas of applications, such as statistics, linear systems, information theory, quantum mechanics, convex analysis, neural networks, and affine differential geometry. The book can serve as a suitable text for a topics course for advanced undergraduates and graduate students.

Understanding Machine Learning

Mathematical Theory of Bayesian Statistics

Generalized Principal Component Analysis

From Geometry, to Physics, to Machine Learning

Algebraic and Geometric Methods in Discrete Mathematics

Sure to be influential, Watanabe s book lays the foundations for the use of algebraic geometry in statistical learning theory. Many models/machines are singular: mixture models, neural networks, HMMs, Bayesian networks, stochastic context-free grammars are major examples. The theory achieved here underpins accurate estimation techniques in the presence of singularities.

This book provides a comprehensive introduction to the latest advances in the mathematical theory and computational tools for modeling high-dimensional data drawn from one or multiple low-dimensional subspaces (or manifolds) and potentially corrupted by noise, gross errors, or outliers. This challenging task requires the development of new algebraic, geometric, statistical, and computational methods for efficient and robust estimation and segmentation of one or multiple subspaces. The book also presents interesting real-world applications of these new methods in image processing, image and video segmentation, face recognition and clustering, and hybrid system identification etc. This book is intended to serve as a textbook for graduate students and beginning researchers in data science, machine learning, computer vision, image and signal processing, and systems theory. It contains ample illustrations, examples, and exercises and is made largely self-contained with three Appendices which survey basic concepts and principles from statistics, optimization, and algebraic-geometry used in this book. René Vidal is a Professor of Biomedical Engineering and Director of the Vision Dynamics and Learning Lab at The Johns Hopkins University. Yi Ma is Executive Dean and Professor at the School of Information Science and Technology at ShanghaiTech University. S. Shankar Sastry is Dean of the College of Engineering, Professor of Electrical Engineering and Computer Science and Professor of Bioengineering at the University of California, Berkeley.

A classic problem in mathematics is solving systems of polynomial equations in several unknowns. Today, polynomial models are ubiquitous and widely used across the sciences. They arise in robotics, coding theory, optimization, mathematical biology, computer vision, game theory, statistics, and numerous other areas. This book furnishes a bridge across mathematical disciplines and exposes many facets of systems of polynomial equations. It covers a wide spectrum of mathematical techniques and algorithms, both symbolic and numerical.The set of solutions to a system of polynomial equations is an algebraic variety - the basic object of algebraic geometry. The algorithmic study of algebraic varieties is the central theme of computational algebraic geometry. Exciting recent developments in computer software for geometric calculations have revolutionized the field. Formerly inaccessible problems are now tractable, providing fertile ground for experimentation and conjecture. The first half of the book gives a snapshot of the state of the art of the topic. Familiar themes are covered in the first five chapters, including polynomials in one variable, Grobner bases of zero-dimensional ideals, Newton polytopes and Bernstein's Theorem, multidimensional resultants, and primary decomposition.The second half of the book explores polynomial equations from a variety of novel and unexpected angles. It introduces interdisciplinary connections, discusses highlights of current research, and outlines possible future algorithms. Topics include computation of Nash equilibria in game theory, semidefinite programming and the real Nullstellensatz, the algebraic geometry of statistical models, the piecewise-linear geometry of valuations and amoebas, and the Ehrenpreis-Palamodov theorem on linear partial differential equations with constant coefficients.Throughout the text, there are many hands-on examples and exercises, including short but complete sessions in MapleR, MATLABR, Macaulay 2, Singular, PHCpack, CoCoA, and SOSTools software. These examples will be particularly useful for readers with no background in algebraic geometry or commutative algebra. Within minutes, readers can learn how to type in polynomial equations and actually see some meaningful results on their computer screens. Prerequisites include basic abstract and computational algebra. The book is designed as a text for a graduate course in computational algebra.

The primary goal of this 2003 book is to give a brief introduction to the main ideas of algebraic and geometric invariant theory. It assumes only a minimal background in algebraic geometry, algebra and representation theory. Topics covered include the symbolic method for computation of invariants on the space of homogeneous forms, the problem of finite-generatedness of the algebra of invariants, the theory of covariants and constructions of categorical and geometric quotients. Throughout, the emphasis is on concrete examples which originate in classical algebraic geometry. Based on lectures given at University of Michigan, Harvard University and Seoul National University, the book is written in an accessible style and contains many examples and exercises. A novel feature of the book is a discussion of possible linearizations of actions and the variation of quotients under the change of linearization. Also includes the construction of toric varieties as torus quotients of affine spaces.

Linear Algebra and Its Applications with R

Lectures on Algebraic Statistics

The Geometry of Multivariate Statistics

A Geometric Primer

Algebraic Statistics for Computational Biology

Knowledge updating is a never-ending process and so should be the revision of an effective textbook. The book originally written fifty years ago has, during the intervening period, been revised and reprinted several times. The authors have, however, been thinking, for the last few years that the book needed not only a thorough revision but rather a substantial rewriting. They now take great pleasure in presenting to the readers the twelfth, thoroughly revised and enlarged, Golden Jubilee edition of the book. The subject-matter in the entire book has been re-written in the light of numerous criticisms and suggestions received from the users of the earlier editions in India and abroad. The basis of this revision has been the emergence of new literature on the subject, the constructive feedback from students and teaching fraternity, as well as those changes that have been made in the syllabi and/or the pattern of examination papers of numerous universities.

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An up-to-date account of algebraic statistics and information geometry, which also explores the emerging connections between these two disciplines.

This book, first published in 2005, offers an introduction to the application of algebraic statistics to computational biology.

Semialgebraic Statistics and Latent Tree Models explains how to analyze statistical models with hidden (latent) variables. It takes a systematic, geometric approach to studying the semialgebraic structure of latent tree models.The first part of the book gives a general introduction to key concepts in algebraic statistics, focusing on methods that a

Combinatorics, Matrix Models and Algebraic Geometry

From Theory to Algorithms

The Calabi – Yau Landscape

Complex Projective Varieties

Methods of Information Geometry

A traditional approach to developing multivariate statistical theory is algebraic. Sets of observations are represented by matrices, linear combinations are formed from these matrices by multiplying them by coefficient matrices, and useful statistics are found by imposing various criteria of optimization on these combinations. Matrix algebra is the vehicle for these calculations. A second approach is computational. Since many users find that they do not need to know the mathematical basis of the techniques as long as they have a way to transform data into results, the computation can be done by a package of computer programs that somebody else has written. An approach from this perspective emphasizes how the computer packages are used, and is usually coupled with rules that allow one to extract the most important numbers from the output and interpret them. Useful as both approaches are--particularly when combined--they can overlook an important aspect of multivariate analysis. To apply it correctly, one needs a way to conceptualize the multivariate relationships that exist among variables. This book is designed to help the reader develop a way of thinking about multivariate statistics, as well as to understand in a broader and more intuitive sense what the procedures do and how their results are interpreted. Presenting important procedures of multivariate statistical theory geometrically, the author hopes that this emphasis on the geometry will give the reader a coherent picture into which all the multivariate techniques fit.

Sure to be influential, Watanabe's book lays the foundations for the use of algebraic geometry in statistical learning theory. Many models/machines are singular: mixture models, neural networks, HMMs, Bayesian networks, stochastic context-free grammars are major examples. The theory achieved here underpins accurate estimation techniques in the presence of singularities.

Can artificial intelligence learn mathematics? The question is at the heart of this original monograph bringing together theoretical physics, modern geometry, and data science. The study of Calabi–Yau manifolds lies at an exciting intersection between physics and mathematics. Recently, there has been much activity in applying machine learning to solve otherwise intractable problems, to conjecture new formulae, or to understand the underlying structure of mathematics. In this book, insights from string and quantum field theory are combined with powerful techniques from complex and algebraic geometry, then translated into algorithms with the ultimate aim of deriving new information about Calabi–Yau manifolds. While the motivation comes from mathematical physics, the techniques are purely mathematical and the theme is that of explicit calculations. The reader is guided through the theory and provided with explicit computer code in standard software such as SageMath, Python and Mathematica to gain hands-on experience in applications of artificial intelligence to geometry. Driven by data and written in an informal style, The Calabi–Yau Landscape makes cutting-edge topics in mathematical physics, geometry and machine learning readily accessible to graduate students and beyond. The overriding ambition is to introduce some modern mathematics to the physicist, some modern physics to the mathematician, and machine learning to both.

The aim of this book is to discuss the fundamental ideas which lie behind the statistical theory of learning and generalization. It considers learning as a general problem of function estimation based on empirical data. Omitting proofs and technical details, the author concentrates on discussing the main results of learning theory and their connections to fundamental problems in statistics. This second edition contains three new chapters devoted to further development of the learning theory and SVM techniques. Written in a readable and concise style, the book is intended for statisticians, mathematicians, physicists, and computer scientists.

The Nature of Statistical Learning Theory

Visual Thinking in Mathematics

Levico Terme, Italy 2013, Editors: Sandra Di Rocco, Bernd Sturmfels

Statistical Methods

Algebraic Geometry I

Tropical geometry is a combinatorial shadow of algebraic geometry, offering new polyhedral tools to compute invariants of algebraic varieties. It is based on tropical algebra, where the sum of two numbers is their minimum and the product is their sum. This turns polynomials into piecewise-linear functions, and their zero sets into polyhedral complexes. These tropical varieties retain a surprising amount of information about their classical counterparts. Tropical geometry is a young subject that has undergone a rapid development since the beginning of the 21st century. While establishing itself as an area in its own right, deep connections have been made to many branches of pure and applied mathematics. This book offers a self-contained introduction to tropical geometry, suitable as a course text for beginning graduate students. Proofs are provided for the main results, such as the Fundamental Theorem and the Structure Theorem. Numerous examples and explicit computations illustrate the main concepts. Each of the six chapters concludes with problems that will help the readers to practice their tropical skills, and to gain access to the research literature. This wonderful book will appeal to students and researchers of all stripes: it begins at an undergraduate level and ends with deep connections to toric varieties, compactifications, and degenerations. In between, the authors provide the first complete proofs in book form of many fundamental results in the subject. The pages are sprinkled with illuminating examples, applications, and exercises, and the writing is lucid and meticulous throughout. It is that rare kind of book which will be used equally as an introductory text by students and as a reference for experts. —Matt Baker, Georgia Institute of Technology Tropical geometry is an exciting new field, which requires tools from various parts of mathematics and has connections with many areas. A short definition is given by Maclagan and Sturmfels: “Tropical geometry is a marriage between algebraic and polyhedral geometry”. This wonderful book is a pleasant and rewarding journey through different landscapes, inviting the readers from a day at a beach to the hills of modern algebraic geometry. The authors present building blocks, examples and exercises as well as recent results in tropical geometry, with ingredients from algebra, combinatorics, symbolic computation, polyhedral geometry and algebraic geometry. The volume will appeal both to beginning graduate students willing to enter the field and to researchers, including experts. —Alicia Dickenstein, University of Buenos Aires, Argentina

The fundamental mathematical tools needed to understand machine learning include linear algebra, analytic geometry, matrix decompositions, vector calculus, optimization, probability and statistics. These topics are traditionally taught in disparate courses, making it hard for data science or computer science students, or professionals, to efficiently learn the mathematics. This self-contained textbook bridges the gap between mathematical and machine learning texts, introducing the mathematical concepts with a minimum of prerequisites. It uses these concepts to derive four central machine learning methods: linear regression, principal component analysis, Gaussian mixture models and support vector machines. For students and others with a mathematical background, these derivations provide a starting point to machine learning texts. For those learning the mathematics for the first time, the methods help build intuition and practical experience with applying mathematical concepts. Every chapter includes worked examples and exercises to test understanding. Programming tutorials are offered on the book's web site.

Discover New Methods for Dealing with High-Dimensional Data A sparse statistical model has only a small number of nonzero parameters or weights; therefore, it is much easier to estimate and interpret than a dense model. Statistical Learning with Sparsity: The Lasso and Generalizations presents methods that exploit sparsity to help recover the underlying signal in a set of data. Top experts in this rapidly evolving field, the authors describe the lasso for linear regression and a simple coordinate descent algorithm for its computation. They discuss the application of l1 penalties to generalized linear models and support vector machines, cover generalized penalties such as the elastic net and group lasso, and review numerical methods for optimization. They also present statistical inference methods for fitted (lasso) models, including the bootstrap, Bayesian methods, and recently developed approaches. In addition, the book examines matrix decomposition, sparse multivariate analysis, graphical models, and compressed sensing. It concludes with a survey of theoretical results for the lasso. In this age of big data, the number of features measured on a person or object can be large and might be larger than the number of observations. This book shows how the sparsity assumption allows us to tackle these problems and extract useful and reproducible patterns from big datasets. Data analysts, computer scientists, and theorists will appreciate this thorough and up-to-date treatment of sparse statistical modeling.

Algebraic geometry, central to pure mathematics, has important applications in such fields as engineering, computer science, statistics and computational biology, which exploit the computational algorithms that the theory provides. Users get the full benefit, however, when they know something of the underlying theory, as well as basic procedures and facts. This book is a systematic introduction to the central concepts of algebraic geometry most useful for computation. Written for advanced undergraduate and graduate students in mathematics and researchers in application areas, it focuses on specific examples and restricts development of formalism to what is needed to address these examples. In particular, it introduces the notion of Gröbner bases early on and develops algorithms for almost everything covered. It is based on courses given over the past five years in a large interdisciplinary programme in computational algebraic geometry at Rice University, spanning mathematics, computer science, biomathematics and bioinformatics.

Mathematics for Machine Learning

Information Geometry and Its Applications

Differential Geometry and Statistics

Lectures on Invariant Theory

Elementary introduction to stable maps and quantum cohomology presents the problem of counting rational plane curves Viewpoint is mostly that of enumerative geometry Emphasis is on examples, heuristic discussions, and simple applications to best convey the intuition behind the subject Ideal for self-study, for a mini-course in quantum cohomology, or as a special topics text in a standard course in intersection theory

This is the first comprehensive book on information geometry, written by the founder of the field. It begins with an elementary introduction to dualistic geometry and proceeds to a wide range of applications, covering information science, engineering, and neuroscience. It consists of four parts, which on the whole can be read independently. A manifold with a divergence function is first introduced, leading directly to dualistic structure, the heart of information geometry. This part (Part I) can be apprehended without any knowledge of differential geometry. An intuitive explanation of modern differential geometry then follows in Part II, although the book is for the most part understandable without modern differential geometry. Information geometry of statistical inference, including time series analysis and semiparametric estimation (the Neyman–Scott problem), is demonstrated concisely in Part III. Applications addressed in Part IV include hot current topics in machine learning, signal processing, optimization, and neural networks. The book is interdisciplinary, connecting mathematics, information sciences, physics, and neurosciences, inviting readers to a new world of information and geometry. This book is highly recommended to graduate students and researchers who seek new mathematical methods and tools useful in their own fields.

Introduction to Tropical Geometry