Analytic Functions Of Several Complex Variables Ams Chelsea Publishing

The purpose of this book is to present the classical analytic function theory of several variables as a standard subject in a course of mathematics after learning the elementary materials (sets, general topology, algebra, one complex variable). This includes the essential parts of Grauert–Remmert's two volumes, GL227(236) (Theory of Stein spaces) and GL265 (Coherent analytic sheaves) with a lowering of the level for novice graduate students (here, Grauert's direct image theorem is limited to the case of finite maps).The core of the theory is "Oka's Coherence", found and proved by Kiyoshi Oka. It is indispensable, not only in the study of complex analysis and complex geometry, but also in a large area of modern mathematics. In this book, just after an introductory chapter on holomorphic functions (Chap. 1), we prove Oka's First Coherence Theorem for holomorphic functions in Chap. 2. This defines a unique character of the book compared with other books on this subject, in which the notion of coherence appears much later.The present book, consisting of nine chapters, gives complete treatments of the following items: Coherence of sheaves of holomorphic functions (Chap. 2); Oka–Cartan's Fundamental Theorem (Chap. 4); Coherence of ideal sheaves of complex analytic subsets (Chap. 6); Coherence of the normalization sheaves of complex spaces (Chap. 6); Grauert's Finiteness Theorem (Chaps. 7, 8); Oka's Theorem for Riemann domains (Chap. 8). The theories of sheaf cohomology and domains of holomorphy are also presented (Chaps. 3, 5). Chapter 6 deals with the theory of complex analytic subsets. Chapter 8 is devoted to the applications of formerly obtained results, proving Cartan–Serre's Theorem and Kodaira's Embedding Theorem. In Chap. 9, we discuss the historical development of "Coherence".It is difficult to find a book at this level that treats all of the above subjects in a completely self-contained manner. In the present volume, a number of classical proofs are improved and simplified, so that the contents are easily accessible for beginning graduate students.

A number of monographs of various aspects of complex analysis in several variables have appeared since the first version of this book was published, but none of them uses the analytic techniques based on the solution of the Neumann Problem as the main tool. The additions made in this third, revised edition place additional stress on results where these methods are particularly important. Thus, a section has been added presenting Ehrenpreis' ``fundamental principle'' in full. The local arguments in this section are closely related to the proof of the coherence of the sheaf of germs of functions vanishing on an analytic set. Also added is a discussion of the theorem of Siu on the Lelong numbers of plurisubharmonic functions. Since the L2 techniques are essential in the proof and plurisubharmonic functions play such an important role in this book, it seems natural to discuss their main singularities.

Complex Analysis 2

Complex Analytic Sets

On Some Banach Spaces of Analytic Functions of Several Complex Variables

Functions of Several Variables

The theory of complex analytic sets is part of the modern geometrical theory of functions of several complex variables. A wide circle of problems in multidimensional complex analysis, related to holomorphic functions and maps, can be reformulated in terms of analytic sets. In these reformulations additional phenomena may emerge, while for the proofs new methods are necessary. (As an example we can mention the boundary properties of conformal maps of domains in the plane, which may be studied by means of the boundary properties of the graphs of such maps.) The theory of complex analytic sets is a relatively young branch of complex analysis. Basically, it was developed to fulfill the need of the theory of functions of several complex variables, but for a long time its development was, so to speak, within the framework of algebraic geometry - by analogy with algebraic sets. And although at present the basic methods of the theory of analytic sets are related with analysis and geometry, the foundations of the theory are expounded in the purely algebraic language of ideals in commutative algebras. In the present book I have tried to eliminate this noncorrespondence and to give a geometric exposition of the foundations of the theory of complex analytic sets, using only classical complex analysis and a minimum of algebra (well-known properties of polynomials of one variable). Moreover, it must of course be taken into consideration that algebraic geometry is one of the most important domains of application of the theory of analytic sets, and hence a lot of attention is given in the present book to algebraic sets.

'Kiyoshi Oka, at the beginning of his research, regarded the collection of problems which he encountered in the study of domains of holomorphy as large mountains which separate today and tomorrow. Thus, he believed that there could be no essential progress in analysis without climbing over these mountains...this book is a worthwhile initial step for the reader in order to understand the mathematical world which was created by Kiyoshi Oka' - from the Preface. This book explains results in the theory of functions of several complex variables which were mostly established from the late nineteenth century through the middle of the twentieth century. In the work, the author introduces the mathematical world created by his advisor, Kiyoshi Oka.In this volume, Oka's work is divided into two parts. The first is the study of analytic functions in univalent domains in ${\mathbf C}^n$. Here Oka proved that three concepts are equivalent: domains of holomorphy, holomorphically convex domains, and pseudoconvex domains; and moreover that the Poincare problem, the Cousin problems, and the Runge problem, when stated properly, can be solved in domains of holomorphy satisfying the appropriate conditions.The second part of Oka's work established a method for the study of analytic functions defined in a ramified domain over ${\mathbf C}^n$ in which the branch points are considered as interior points of the domain. Here analytic functions in an analytic space are treated, which is a slight generalization of a ramified domain over ${\mathbf C}^n$. In writing the book, the author's goal was to bring to readers a real understanding of Oka's original papers. This volume is an English translation of the original Japanese edition, published by the University of Tokyo Press (Japan). It would make a suitable course text for advanced graduate level introductions to several complex variables.

Function Theory in Classical Domains Complex Potential Theory

Lectures Delivered at the Institute for Advanced Study, 1948-1949

Functions of Several Complex Variables

The theory of analytic functions of several complex variables enjoyed a period of remarkable development in the middle part of the twentieth century. After initial successes by Poincare and others in the late 19th and early 20th centuries, the theory encountered obstacles that prevented it from growing quickly into an analogue of the theory for functions of one complex variable. Beginning in the 1930s, initially through the work of Oka, then H. Cartan, and continuing with the work of Grauert, Remmert, and others, new tools were introduced into the theory of several complex variables that resolved many of the open problems and fundamentally changed the landscape of the subject. These tools included a central role for sheaf theory and increased uses of topology and algebra. The book by Gunning and Rossi was the first of the

modern era of the theory of several complex variables, which is distinguished by the use of these methods. The intention of Gunning and Rossi's book is to provide an extensive introduction to the Oka-Cartan theory and some of its applications, and to the general theory of analytic spaces. Fundamental concepts and techniques are discussed as early as possible. The first chapter covers material suitable for a one-semester graduate course, presenting many of the central problems and techniques, often in special cases. The later chapters give more detailed expositions of sheaf theory for analytic functions and the theory of complex analytic spaces. Since its original publication, this book has become a classic resource for the modern approach to functions of several complex variables and the theory of analytic spaces. Further information about this book, including updates, can be found at the following URL: www.ams.org/bookpages/chel-368.

Authored by a ranking authority in harmonic analysis of several complex variables, this book embodies a state-of-the-art entrée at the intersection of two important fields of research: complex analysis and harmonic analysis. Written with the graduate student in mind, it is assumed that the reader has familiarity with the basics of complex analysis of one and several complex variables as well as with real and functional analysis. The monograph is largely self-contained and develops the harmonic analysis of several complex variables from the first principles. The text includes copious examples, explanations, an exhaustive bibliography for further reading, and figures that illustrate the geometric nature of the subject. Each chapter ends with an exercise set. Additionally, each chapter begins with a prologue, introducing the reader to the subject matter that follows; capsules presented in each section give perspective and a spirited launch to the segment; preludes help put ideas into context. Mathematicians and researchers in several applied disciplines will find the breadth and depth of the treatment of the subject highly useful.

Lectures Delivered at the Institute for Advanced Study During the Fall Term of 1948

Analytic Function of Several Complex Variables

Function Theory in Several Complex Variables

A Dirichlet Principle for Analytic Functions of Several Complex Variables

Plurisubharmonic functions playa major role in the theory of functions of several complex variables. The extensiveness of plurisubharmonic functions, the simplicity of their definition together with the richness of their properties and. most importantly, their close connection with holomorphic functions have assured plurisubharmonic functions a lasting place in multidimensional complex analysis. (Pluri)subharmonic functions first made their appearance in the works of Hartogs at the beginning of the century. They figure in an essential way, for example, in the proof of the famous theorem of Hartogs (1906) on joint holomorphicity. Defined at first on the complex plane IC, the class of subharmonic functions became thereafter

one of the most fundamental tools in the investigation of analytic functions of one or several variables. The theory of subharmonic functions was developed and generalized in various directions: subharmonic functions in Euclidean space IRn, plurisubharmonic functions in complex space en and others. Subharmonic functions and the foundations ofthe associated classical poten tial theory are sufficiently well exposed in the literature, and so we introduce here only a few fundamental results which we require. More detailed expositions can be found in the monographs of Privalov (1937), Brelot (1961), and Landkof (1966). See also Brelot (1972), where a history of the development of the theory of subharmonic functions is given.

This second edition presents a collection of exercises on the theory of analytic functions, including completed and detailed solutions. It introduces students to various applications and aspects of the theory of analytic functions not always touched on in a first course, while also addressing topics of interest to electrical engineering students (e.g., the realization of rational functions and its connections to the theory of linear systems and state space representations of such systems). It provides examples of important Hilbert spaces of analytic functions (in particular the Hardy space and the Fock space), and also includes a section reviewing essential aspects of topology, functional analysis and Lebesgue integration. Benefits of the 2nd edition Rational functions are now covered in a separate chapter. Further, the section on conformal mappings has been expanded.

Analytic Function Theory of Several Variables

Complex Variables and Analytic Functions: An Illustrated Introduction

Theory of analytic functions of several complex variables

Lectures Delivered at the Institute for Advanced Study 1948 - 1949

This book is a polished version of my course notes for Math 6283, Several Complex Variables, given in Spring 2014 and Spring 2016 semester at Oklahoma State University. The course covers basics of holomorphic function theory, CR geometry, the dbar problem, integral kernels and basic theory of complex analytic subvarieties. See http: //www.jirka.org/scv/ for more information.

In this textbook, a concise approach to complex analysis of one and several variables is presented. After an introduction of Cauchy‘s integral theorem general versions of Runge‘s approximation theorem and Mittag-Leffler‘s theorem are discussed. The fi rst part ends with an analytic characterization of simply connected domains. The second part is concerned with functional analytic methods: Fréchet and Hilbert spaces of holomorphic functions, the Bergman kernel, and unbounded operators on Hilbert spaces to tackle the theory of several variables, in particular the inhomogeneous Cauchy-Riemann equations and the d-bar Neumann operator. Contents Complex numbers and functions Cauchy's Theorem and Cauchy's formula Analytic continuation Construction and approximation of holomorphic functions Harmonic functions Several complex variables Bergman spaces The canonical solution operator to Nuclear Fréchet spaces of holomorphic functions The -complex The twisted -complex and Schrödinger operators

Several Complex Variables II

Complex Analysis

Analytic functions of several complex variables

Elements of Oka's Coherence

At almost all academic institutions worldwide, complex variables and analytic functions are utilized in courses on applied mathematics, physics, engineering, and other related subjects. For most students, formulas alone do not provide a sufficient introduction to this widely taught material, yet illustrations of functions are sparse in current books on the topic. This is the first primary introductory textbook on complex variables and analytic functions to make extensive use of functional illustrations. Aiming to reach undergraduate students entering the world of complex variables and analytic functions, this book utilizes graphics to visually build on familiar cases and illustrate how these same functions extend beyond the real axis. It covers several important topics that are omitted in nearly all recent texts, including techniques for analytic continuation and discussions of elliptic functions and of Wiener–Hopf methods. It also presents current advances in research, highlighting the subject's active and fascinating frontier. The primary audience for this textbook is undergraduate students taking an introductory course on complex variables and analytic functions. It is also geared toward graduate students taking a second semester course on these topics, engineers and physicists who use complex variables in their work, and students and researchers at any level who want a reference book on the subject.

Since the 1960s, there has been a flowering in higher-dimensional complex analysis. Both classical and new results in this area have found numerous applications in analysis, differential and algebraic geometry, and, in particular, contemporary mathematical physics. In many areas of modern mathematics, the mastery of the foundations of higher-dimensional complex analysis has become necessary for any specialist. Intended as a first study of higher-dimensional complex analysis, the book covers the theory of holomorphic functions of several complex variables, holomorphic mappings, and submanifolds of complex Euclidean space.

Introduction to Complex Analysis

Introduction to the Theory of Analytic Functions of Several Complex Variables

Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains

Applied Complex Variables

This highly regarded text is directed toward advanced undergraduates and graduate students in mathematics who are interested in developing a firm foundation in the theory of functions of a complex variable. The treatment departs from traditional presentations in its early development of a rigorous discussion of the theory of multiple-valued analytic functions on the basis of analytic continuation. Thus it offers an early introduction of Riemann surfaces,

conformal mapping, and the applications of residue theory. M. A. Evgrafov focuses on aspects of the theory that relate to modern research and assumes an acquaintance with the basics of mathematical analysis derived from a year of advanced calculus. Starting with an introductory chapter containing the fundamental results concerning limits, continuity, and integrals, the book addresses analytic functions and their properties, multiple-valued analytic functions, singular points and expansion in series, the Laplace transform, harmonic and subharmonic functions, extremal problems and distribution of values, and other subjects. Chapters are largely self-contained, making this volume equally suitable for the classroom or independent study.
Fundamentals of analytic function theory — plus lucid exposition of 5 important applications: potential theory, ordinary differential equations, Fourier transforms, Laplace transforms, and asymptotic expansions. Includes 66 figures.

Bounded Analytic Functions

Tasty Bits of Several Complex Variables

An Introduction to Complex Analysis in Several Variables

Analytic Functions

I - Entire functions of several complex variables constitute an important and original chapter in complex analysis. The study is often motivated by certain applications to specific problems in other areas of mathematics: partial differential equations via the Fourier-Laplace transformation and convolution operators, analytic number theory and problems of transcen dence, or approximation theory, just to name a few. What is important for these applications is to find solutions which satisfy certain growth conditions. The specific problem defines inherently a growth scale, and one seeks a solution of the problem which satisfies certain growth conditions on this scale, and sometimes solutions of minimal asymp totic growth or optimal solutions in some sense. For one complex variable the study of solutions with growth conditions forms the core of the classical theory of entire functions and, historically, the relationship between the number of zeros of an entire function f(z) of one complex variable and the growth of If I (or equivalently log If I) was the first example of a systematic study of growth conditions in a general setting. Problems with growth conditions on the solutions demand much more precise information than existence theorems. The correspondence between two scales of growth can be interpreted often as a correspondence between families of bounded sets in certain Frechet spaces. However, for applications it is of utmost importance to develop precise and explicit representations of the solutions.

The book contains a complete self-contained introduction to highlights of classical complex analysis. New proofs and some new results are included. All needed notions are developed within the book: with the exception of some basic facts which can be found in the¯ rst volume. There is no comparable treatment in the literature.

Theory of Analytic Functions of Several Complex Variables

A Complex Analysis Problem Book

Analytic Functions of Several Complex Variables

A Functional Analytic Approach

Elementary Theory of Analytic Functions of One or Several Complex VariablesCourier Corporation

Basic treatment includes existence theorem for solutions of differential systems where data is analytic, holomorphic functions, Cauchy's integral, Taylor and Laurent expansions, more. Exercises. 1973 edition.

Harmonic and Complex Analysis in Several Variables

Analytic Functions of Several Complex Variables [by] Robert C. Gunning [and] Hugo Rossi

Translated from the Russian by A. A. Brown, J. M. Danskin, and E. Hewitt

Special Chapters in the Theory of Analytic Functions of Several Complex Variables

An Introduction to Complex Analysis in Several Variables

This book is an account of the theory of Hardy spaces in one dimension, with emphasis on some of the exciting developments of the past two decades or so. The last seven of the ten chapters are devoted in the main to these recent developments. The motif of the theory of Hardy spaces is the interplay between real, complex, and abstract analysis. While paying proper attention to each of the three aspects, the author has underscored the effectiveness of the methods coming from real analysis, many of them developed as part of a program to extend the theory to Euclidean spaces, where the complex methods are not available.

Elementary Theory of Analytic Functions of One or Several Complex Variables

Entire Functions of Several Complex Variables

Riemann Surfaces, Several Complex Variables, Abelian Functions, Higher Modular Functions