Linear And Quasilinear Parabolic Problems Volume I Abstract Linear Theory Monographs In Mathematics V 1

This book unifies the different approaches in studying elliptic and parabolic partial differential equations with discontinuous coefficients. To the enlarging market of researchers in applied sciences, mathematics and physics, it gives concrete answers to questions suggested by non-linear models. Providing an up-to date survey on the results concerning elliptic and parabolic operators on a high level, the authors serve the reader in doing further research. Being themselves active researchers in the field, the authors describe both on the level of good examples and precise analysis, the crucial role played by such requirements on the coefficients as the Cordes condition, Campanato's nearness condition, and vanishing mean oscillation condition. They present the newest results on the basic boundary value problems for operators with VMO coefficients and non-linear operators with discontinuous coefficients and state a lot of open problems in the field.

In this volume boundary value problems are studied from two points of view; solvability, unique or otherwise, and the effect of various smoothness properties of the given functions on the smoothness of the solutions. There are seven chapters contained in this volume. Chapter One gives a statement of the new results and an historical sketch. Chapter two introduces the various function spaces typical of modern Russian-style functional analysis. Chapters three and four deal with linear equations. Chapter six concerns itself with quasilinear equations, and chapter seven with systems of equations. These last four chapters can be read independently of one another.

This volume discusses an in-depth theory of function spaces in an Euclidean setting, including several new features, not previously covered in the literature. In particular, it develops a unified theory of anisotropic Besov and Bessel potential spaces on Euclidean corners, with infinite-dimensional Banach spaces as targets. It especially highlights the most important subclasses of Besov spaces, namely Slobodeckii and Hölder spaces. In this case, no restrictions are imposed on the target spaces, except for reflexivity assumptions in duality results. In this general setting, the author proves sharp embedding, interpolation, and trace theorems, point-wise multiplier results, as well as Gagliardo-Nirenberg estimates and generalizations of Aubin-Lions compactness theorems. The results presented pave the way for new applications in situations where infinite-dimensional target spaces are relevant – in the realm of stochastic differential equations, for example.

This reference - based on the Conference on Differential Equations, held in Bologna - provides information on current research in parabolic and hyperbolic differential equations. Presenting methods and results in semigroup theory and their applications to evolution equations, this book focuses on topics including: abstract parabolic and hyperbolic linear differential equations; nonlinear abstract parabolic equations; holomorphic semigroups; and Volterra operator integral equations.;With contributions from international experts, Differential Equations in Banach Spaces is intended for research mathematicians in functional analysis, partial differential equations, operator theory and control theory; and students in these disciplines.

Differential Equations in Banach Spaces

Volume I: Abstract Linear Theory

Qualitative Properties of Solutions

The Linearly Implicit Euler Method for Quasi-linear Parabolic Differential Equations

Elliptic and Parabolic Equations with Discontinuous Coefficients

This monograph presents a systematic theory of weak solutions in Hilbert-Sobolev spaces of initial-boundary value problems for parabolic systems of partial differential equations with general essential and natural boundary conditions and minimal hypotheses on coefficients. Applications to quasilinear systems are given, including local existence for large data, global existence near an attractor, the Leray and Hopf theorems for the Navier-Stokes equations and results concerning invariant regions. Supplementary material is provided, including a self-contained treatment of the calculus of Sobolev functions on the boundaries of Lipschitz domains and a thorough discussion of measurability considerations for elements of Bochner-Sobolev spaces. This book will be particularly useful both for researchers requiring accessible and broadly applicable formulations of standard results as well as for students preparing for research in applied analysis. Readers should be familiar with the basic facts of measure theory and functional analysis, including weak derivatives and Sobolev spaces. Prior work in partial differential equations is helpful but not required.

The present volume is dedicated to celebrate the work of the renowned mathematician Herbert Amann, who had a significant and decisive influence in shaping Nonlinear Analysis. Most articles published in this book, which consists of 32 articles in total, written by highly distinguished researchers, are in one way or another related to the scientific works of Herbert Amann. The contributions cover a wide range of nonlinear elliptic and parabolic equations with applications to natural sciences and engineering. Special topics are fluid dynamics, reaction-diffusion systems, bifurcation theory, maximal regularity, evolution equations, and the theory of function spaces.

Our research concerns undetermined coefficient problems in partial differential equations, in particular those problems where the unknown coefficients depend only on the dependent variables. The problems modeled by these equations are related to the determination of unknown physical laws or relationships. The nonlinear terms which we seek to recover in our model problems correspond to material properties that have physical significance. These include temperature dependent specific heats, conductivities, and

reaction terms. The main analytical tool is the Fixed Point Projection method, which the investigators have developed for use in elliptic and parabolic inverse problems. This method involves projecting the underlying differential operator onto that subset of the domain where the overposed data is given, and reformulating the inverse problem as an equivalent fixed point problem, which is then solved by iteration. Analytical as well as numerical results have been obtained by the investigators, and the FPP method is currently being extended to hyperbolic inverse problems. (kr).

Introduction. Maximum principles. Introduction to the theory of weak solutions. Hölder estimates. Existence, uniqueness, and regularity of solutions. Further theory of weak solutions. Strong solutions. Fixed point theorems and their applications. Comparison and maximum principles. Boundary gradient estimates. Global and local gradient bounds. Hölder gradient estimates and existence theorems. The oblique derivative problem for quasilinear parabolic equations. Fully nonlinear equations. Introduction. Monge-Ampère and Hessian equations.

Global Uniqueness, Global Convergence and Experimental Data

Second Order Parabolic Differential Equations

Nonlinear Parabolic Equations and Hyperbolic-Parabolic Coupled Systems

Abstract Linear Theory

Linear and Quasi-linear Evolution Equations in Hilbert Spaces

This Research Note presents some recent advances in various important domains of partial differential equations and applied mathematics including equations and systems of elliptic and parabolic type and various applications in physics, mechanics and engineering. These topics are now part of various areas of science and have experienced tremendous development during the last decades. ------------------------------------

This treatise gives an exposition of the functional analytical approach to quasilinear parabolic evolution equations, developed to a large extent by the author during the last 10 years. This approach is based on the theory of linear nonautonomous parabolic evolution equations and on interpolation-extrapolation techniques. It is the only general method that applies to noncoercive quasilinear parabolic systems under nonlinear boundary conditions. The present first volume is devoted to a detailed study of nonautonomous linear parabolic evolution equations in general Banach spaces. It contains a careful exposition of the constant domain case, leading to some improvements of the classical Sobolevskii-Tanabe results. It also includes recent results for equations possessing constant interpolation spaces. In addition, systematic presentations of the theory of maximal regularity in spaces of continuous and Hölder continuous functions, and in Lebesgue spaces, are given. It includes related recent theorems in the field of harmonic analysis in Banach spaces and on operators possessing bounded imaginary powers. Lastly, there is a complete presentation of the technique of interpolation-extrapolation spaces and of evolution equations in those spaces, containing many new results.

The smoothness of solutions for quasilinear systems is one of the most important problems in modern mathematical physics. This book deals with regular or strong solutions for general quasilinear second-order elliptic and parabolic systems. Applications in solid mechanics, hydrodynamics, elasticity and plasticity are described. The results presented are based on two main ideas: the universal iterative method, and explicit, sometimes sharp, coercivity estimates in weighted spaces. Readers are assumed to have a standard background in analysis and PDEs.

Abstract: "Following the method of lines approach parabolic problems discretized in space by any usual method are discretized in time by the linearly implicit Euler method. To avoid order reduction occurring for problems with time dependent boundary conditions a modification of the method is proposed. This modified method is proved to be of uniform (i.e. independently of the space discretization) order 1 of covergence for wide classes of semi-linear and quasi-linear problems."

Pont-A-Mousson 1994

Regularity Problem for Quasilinear Elliptic and Parabolic Systems

Elliptic & Parabolic Equations

Theory and Implementation

Undetermined Coefficient Problems for Quasi-Linear Parabolic Equations

Focuses on three primal DG methods, covering both theory and computation, and providing the basic tools for analysis.

This book is devoted to the qualitative study of solutions of superlinear elliptic and parabolic partial differential equations and systems. This class of problems contains, in particular, a number of reaction-diffusion systems which arise in various mathematical models, especially in chemistry, physics and biology. The book is self-contained and up-to-date, taking special care on the didactical preparation of the material. is devoted to problems that are intensively studied but have not been treated thus far in depth in the book literature.

This book summarizes the main analytical and numerical results of Carleman estimates. In the analytical part, Carleman estimates for thre main types of Partial Differential Equations (PDEs) are derived. In the numerical part, first numerical methods are proposed to solve ill-pose Cauchy problems for both linear and quasilinear PDEs. Next, various versions of the convexification method are developed for a number o Coefficient Inverse Problems.

This book details the mathematical developments in total variation based image restauration. From the reviews: "This book is devoted to P of elliptic and parabolic type associated to functionals having a linear growth in the gradient, with a special emphasis on the applications related to image restoration and nonlinear filters....The book is written with great care, paying also a lot of attention to the bibliographical an historical notes."-- ZENTRALBLATT MATH

Volume II: Function Spaces

The Herbert Amann Festschrift

Moving Interfaces and Quasilinear Parabolic Evolution Equations

Inverse Problems and Carleman Estimates

Linear and Quasilinear Parabolic Systems: Sobolev Space Theory

This monograph is devoted to the global existence, uniqueness and asymptotic behaviour of smooth solutions to both initial value problems and initial boundary value problems for nonlinear parabolic equations and hyperbolic parabolic coupled systems. Most of the material is based on recent research carried out by the author and his collaborators. The book can be divided into two parts. In the first part, the results on decay of solutions to nonlinear parabolic equations and hyperbolic parabolic coupled systems are obtained, and a chapter is devoted to the global existence of small smooth solutions to fully nonlinear parabolic equations and quasilinear hyperbolic parabolic coupled systems. Applications of the results to nonlinear thermoelasticity and fluid dynamics are also shown. Some nonlinear parabolic equations and coupled systems arising from the study of phase transitions are investigated in the second part of the book. The global existence, uniqueness and asymptotic behaviour of smooth solutions with arbitrary initial data are obtained. The final chapter is further devoted to related topics: multiplicity of equilibria and the existence of a global attractor, inertial manifold and inertial set. A knowledge of partial differential equations and Sobolev spaces is assumed. As an aid to the reader, the related concepts and results are collected and the relevant references given in the first chapter. The work will be of interest to researchers and graduate students in pure and applied mathematics, mathematical physics and applied sciences.

The book shows how the abstract methods of analytic semigroups and evolution equations in Banach spaces can be fruitfully applied to the study of parabolic problems. Particular attention is paid to optimal regularity results in linear equations. Furthermore, these results are used to study several other problems, especially fully nonlinear ones. Owing to the new unified approach chosen, known theorems are presented from a novel perspective and new results are derived. The book is self-contained. It is addressed to PhD students and researchers interested in abstract evolution equations and in parabolic partial differential equations and systems. It gives a comprehensive overview on the present state of the art in the field, teaching at the same time how to exploit its basic techniques. - - - This very interesting book provides a systematic treatment of the basic theory of analytic semigroups and abstract parabolic equations in general Banach spaces, and how this theory may be used in the study of parabolic partial differential equations; it takes into account the developments of the theory during the last fifteen years. (...) For instance, optimal regularity results are a typical feature of abstract parabolic equations; they are comprehensively studied in this book, and yield new and old regularity results for parabolic partial differential equations and systems. (Mathematical Reviews) Motivated by applications to fully nonlinear problems the approach is focused on classical solutions with continuous or Hölder continuous derivatives. (Zentralblatt MATH)

The objectives of this monograph are to present some topics from the theory of monotone operators and nonlinear semigroup theory which are directly applicable to the existence and uniqueness theory of initial-boundary-value problems for partial differential equations and to construct such operators as realizations of those problems in appropriate function spaces. A highlight of this presentation is the large number and variety of examples introduced to illustrate the connection between the theory of nonlinear operators and partial differential equations. These include primarily semilinear or quasilinear equations of elliptic or of parabolic type, degenerate cases with change of type, related systems and variational inequalities, and spatial boundary conditions of the usual Dirichlet, Neumann, Robin or dynamic type. The discussions of evolution equations include the usual initial-value problems as well as periodic or more general nonlocal constraints, history-value problems, those which may change type due to a possibly vanishing coefficient of the time derivative, and other implicit evolution equations or systems including hysteresis models. The scalar conservation law and semilinear wave equations are briefly mentioned, and hyperbolic systems arising from vibrations of elastic-plastic rods are developed. The origins of a representative sample of such problems are given in the appendix.

In this monograph, the authors develop a comprehensive approach for the mathematical analysis of a wide array of problems involving moving interfaces. It includes an in-depth study of abstract quasilinear parabolic evolution equations, elliptic and parabolic boundary value problems, transmission problems, one- and two-phase Stokes problems, and the equations of incompressible viscous one- and two-phase fluid flows. The theory of maximal regularity, an essential element, is also fully developed. The authors present a modern approach based on powerful tools in classical analysis, functional analysis, and vector-valued harmonic analysis. The theory is applied to problems in two-phase fluid dynamics and phase transitions, one-phase generalized Newtonian fluids, nematic liquid crystal flows, Maxwell-Stefan diffusion, and a variety of geometric evolution equations. The book also includes a discussion of the underlying physical and thermodynamic principles governing the equations of fluid flows and phase transitions, and an exposition of the geometry of moving hypersurfaces.

Nonlinear Parabolic Equations

Degenerate Parabolic Equations

Linear and Quasi-linear Equations of Parabolic Type

On the Initial Value Problem for the Quasi-linear Parabolic Partial Differential Equation

Linear And Nonlinear Parabolic Complex Equations

This book considers evolution equations of hyperbolic and parabolic type. These equations are studied from a common point of view, using elementary methods, such as that of energy estimates, which prove to be quite versatile. The authors emphasize the Cauchy problem and present a unified theory for the treatment of these equations. In particular, they provide local and global existence results, as well as strong well-posedness and asymptotic behavior results for the Cauchy problem for quasi-linear equations. Solutions of linear equations are constructed explicitly, using the Galerkin method; the linear theory is then applied to quasi-linear equations, by means of a linearization and fixed-point technique. The authors also compare hyperbolic and parabolic problems, both in terms of singular perturbations, on compact time intervals, and asymptotically, in terms of the diffusion phenomenon, with new results on decay estimates for strong solutions of homogeneous quasi-linear equations of each type. This textbook presents a valuable introduction to topics in the theory of evolution equations, suitable for advanced graduate students. The exposition is largely self-contained. The initial chapter reviews the essential material from functional analysis. New ideas are introduced along with their context. Proofs are detailed and carefully presented. The book concludes with a chapter on applications of the theory to Maxwell's equations and von Karman's equations.

Evolved from the author's lectures at the University of Bonn's Institut für angewandte Mathematik, this book reviews recent progress toward understanding of the local structure of solutions of degenerate and singular *parabolic partial differential equations.*

This book provides an introduction to elliptic and parabolic equations. While there are numerous monographs focusing separately on each kind of equations, there are very few books treating these two kinds of equations in combination. This book presents the related basic theories and methods to enable readers to appreciate the commonalities between these two kinds of equations as well as contrast the similarities and differences between them.

In this treatise we present the semigroup approach to quasilinear evolution equa of parabolic type that has been developed over the last ten years, approxi tions mately. It emphasizes the dynamic viewpoint and is sufficiently general and flexible to encompass a great variety of concrete systems of partial differential equations occurring in science, some of those being of rather 'nonstandard' type. In partic ular, to date it is the only general method that applies to noncoercive systems. Although we are interested in nonlinear problems, our method is based on the theory of linear holomorphic semigroups. This distinguishes it from the theory of nonlinear contraction semigroups whose basis is a nonlinear version of the Hille Yosida theorem: the Crandall-Liggett theorem. The latter theory is well-known and well-documented in the literature. Even though it is a powerful technique having found many applications, it is limited in its scope by the fact that, in concrete applications, it is closely tied to the maximum principle. Thus the theory of nonlinear contraction semigroups does not apply to systems, in general, since they do not allow for a maximum principle. For these reasons we do not include that theory.

Parabolic Problems

Linear Discrete Parabolic Problems

Strong solutions for quasi-linear elliptic-parabolic problems with time-dependent obstacles

Recent Advances in Elliptic and Parabolic Problems

The book is an account on recent advances in elliptic and parabolic problems and related equations, including general quasi-linear equations, variational structures, Bose-Einstein condensate, Chern–Simons model, geometric shell theory and stability in fluids. It presents very up-to-date research on central issues of these problems such as maximal regularity, bubbling, blowing-up, bifurcation of solutions and wave interaction. The contributors are well known leading mathematicians and prominent young researchers. The proceedings have been selected for coverage in: • Index to Scientific & Technical Proceedings® (ISTP® / ISI Proceedings) • Index to Scientific & Technical Proceedings (ISTP CDROM version / ISI Proceedings) • CC Proceedings — Engineering & Physical Sciences Contents:Maximal Regularity and Quasilinear Parabolic Boundary Value Problems (H Amann)Remarks on the Two and Three Membranes Problem (J-F Rodrigues et al.)Bubbling and Criticality in Two and Higher Dimensions (M del Pino & M Musso)Blow Up Solutions for a Liouville Equation with Singular Data (P Esposito)Problems in Unbounded Cylindrical Domains (P Guidotti)Entire Solutions of Some Reaction–Diffusion on Equations (J-S Guo)Some Abelian Gauge Field Theories in the Self-dual and Nonself-dual Cases (J Han & N Kim)Ginzburg–Landau Equations on Non-uniform Media (S Kosugi)Finding the Elasticae by Means of Geometric Gradient Flows (C-C Lin & H R Schwetlick)Free Work Identity and Nonlinear Instability in Fluids with Free Boundaries (M Padula)Complete and Energy Blow-up in Superlinear Parabolic Problems (P Quittner)Non-stabilizing Solutions for a Supercritical Semilinear Parabolic Equation (E Yanagida)and other papers Readership: Graduate students and researchers in partial differential equations and mathematical physics. Keywords:Elliptic Equations;Parabolic Problems;Nonlinear Analysis;Partial Differential EquationsKey Features:Presents up-to-date research in many important and hot topicsWritten by first class researchers in related fieldsContains rich models arising from different fields

This volume introduces a unified, self-contained study of linear discrete parabolic problems through reducing the starting discrete problem to the Cauchy problem for an evolution equation in discrete time. Accessible to beginning graduate students, the book contains a general stability theory of discrete evolution equations in Banach space and gives applications of this theory to the analysis of various classes of modern discretization methods, among others, Runge-Kutta and linear multistep methods as well as operator splitting methods. Key features: * Presents a unified approach to examining discretization methods for parabolic equations. * Highlights a stability theory of discrete evolution equations (discrete semigroups) in Banach space. * Deals with both autonomous and non-autonomous equations as well as with equations with memory. * Offers a series of numerous well-posedness and convergence results for various discretization methods as applied to abstract parabolic equations; among others, Runge-Kutta and linear multistep methods as well as certain operator splitting methods. * Provides comments of results and historical remarks after each chapter. · Presents a unified approach to examining discretization methods for parabolic equations. · Highlights a stability theory of discrete evolution equations (discrete semigroups) in Banach space. · Deals with both autonomous and non-autonomous equations as well as with equations with memory. · Offers a series of numerous well-posedness and convergence results for various discretization

methods as applied to abstract parabolic equations; among others, Runge-Kutta and linear multistep methods as well as certain operator splitting methods as well as certain operator splitting methods are studied in detail. ·Provides comments of results and historical remarks after each chapter.

The volume originates from the 'Conference on Nonlinear Parabolic Problems' held in celebration of Herbert Amann's 70th birthday at the Banach Center in Bedlewo, Poland. It features a collection of peer-reviewed research papers by recognized experts highlighting recent advances in fields of Herbert Amann's interest such as nonlinear evolution equations, fluid dynamics, quasi-linear parabolic equations and systems, functional analysis, and more.

Linear and Quasilinear Parabolic ProblemsVolume I: Abstract Linear TheorySpringer Science & Business Media

Superlinear Parabolic Problems

Linear and Quasilinear Parabolic Problems

Qualitative Theory of Parabolic Equations

Discontinuous Galerkin Methods for Solving Elliptic and Parabolic Equations

Elliptic and Parabolic Problems

Blow-up for Higher-Order Parabolic, Hyperbolic, Dispersion and Schrödinger Equations shows how four types of higher-order nonlinear evolution partial differential equations (PDEs) have many commonalities through their special quasilinear degenerate representations. The authors present a unified approach to deal with these quasilinear PDEs. The book first studies the particular self-similar singularity solutions (patterns) of the equations. This approach allows four different classes of nonlinear PDEs to be treated simultaneously to establish their striking common features. The book describes many properties of the equations and examines traditional questions of existence/nonexistence, uniqueness/nonuniqueness, global asymptotics, regularizations, shock-wave theory, and various blow-up singularities. Preparing readers for more advanced mathematical PDE analysis, the book demonstrates that quasilinear degenerate higher-order PDEs, even exotic and awkward ones, are not as daunting as they first appear. It also illustrates the deep features shared by several types of nonlinear PDEs and encourages readers to develop further this unifying PDE approach from other viewpoints.

This book deals mainly with linear and nonlinear parabolic equations and systems of second order. It first transforms the real forms of parabolic equations and systems into complex forms, and then discusses several initial boundary value problems and Cauchy problems for quasilinear and nonlinear parabolic complex equations of second order with smooth coefficients or measurable coefficients. Parabolic complex equations are discussed in the nonlinear case and the boundary conditions usually include the initial irregular oblique derivative. The boundary value problems are considered in multiply connected domains and several methods are used.

This book is devoted to the qualitative study of solutions of superlinear elliptic and parabolic partial differential equations and systems. This class of problems contains, in particular, a number of reaction-diffusion systems which arise in various mathematical models, especially in chemistry, physics and biology. The first two chapters introduce to the field and enable the reader to get acquainted with the main ideas by studying simple model problems, respectively of elliptic and parabolic type. The subsequent three chapters are devoted to problems with more complex structure; namely, elliptic and parabolic systems, equations with gradient depending nonlinearities, and nonlocal equations. They include many developments which reflect several aspects of current research. Although the techniques introduced in the first two chapters provide efficient tools to attack some aspects of these problems, they often display new phenomena and specifically different behaviors, whose study requires new ideas. Many open problems are mentioned and commented. The book is self-contained and up-to-date, it has a high didactic quality. It is devoted to problems that are intensively studied but have not been treated so far in depth in the book literature. The intended audience includes graduate and postgraduate students and researchers working in the field of partial differential equations and applied mathematics. The first edition of this book has become one of the standard references in the field. This second edition provides a revised text and contains a number of updates reflecting significant recent advances that have appeared in this growing field since the first edition.

A Special Tribute to the Work of Herbert Amann

Monotone Operators in Banach Space and Nonlinear Partial Differential Equations

Blow-up, Global Existence and Steady States

Parabolic Quasilinear Equations Minimizing Linear Growth Functionals

Nonlinear Elliptic and Parabolic Problems