Multiplicative Number Theory I Classical Theory

Several years ago I was invited to an American university to give

one-term graduate course on Siegel modular forms, Hecke operators, and related zeta functions. The idea to present in a concise but basically complete and self-contained form an int- duction to an important and

developing area based partly on my own work attracted me. I accepted the invitation and started to prepare the course. Unfortunately, the visit was not realized. But the idea of such a course continued to be alive till

after a number of years this book was ?nally completed. I hope that this short book will serve to attract young researchers to this beautiful ?eld, and that it will simplify and make more pleasant the initial steps. No special

knowledge is presupposed for reading this book beyond standard courses in algebra and calculus (one and several variables), although some skill in working with mathematical texts would be helpful. The reader will

judge whether the result was worth the effort. Dedications. The ideas of Goro Shimura exerted a deep in?uence on the number theory of the second half of the twentieth century in general and on the author's formation in

particular. When Andre ` Weil was signing a copy of his “Basic Number Theory” to my son, he wrote in Russian, ”To Fedor Anatolievich hoping that he will become a number theoretist”. Fedor has chosen computer

science. Now I pass on the idea to Fedor’s daughter, Alexandra Fedorovna.
Developed from the author's popular text, A Concise Introduction to the Theory of Numbers, this book provides a

comprehensive initiation to all the major branches of number theory. Beginning with the rudiments of the subject, the author proceeds to more advanced topics, including elements of cryptography and primality

testing, an account of number fields in the classical vein including properties of their units, ideals and ideal classes, aspects of analytic number theory including studies of the Riemann zeta-function, the prime-number

theorem and primes in arithmetical progressions, a description of the Hardy–Littlewood and sieve methods from respectively additive and multiplicative number theory and an exposition of the

arithmetic of elliptic curves. The book includes many worked examples, exercises and further reading. Its wider coverage and versatility make this book suitable for courses extending from the elementary to beginning graduate

studies.
Elements of the Theory of Numbers teaches students how to develop, implement, and test numerical methods for standard mathematical problems. The authors have created a two-

pronged pedagogical approach that integrates analysis and algebra with classical number theory. Making greater use of the language and concepts in algebra and analysis than is traditionally encountered in introductory

courses, this pedagogical approach helps to instill in the minds of the students the idea of the unity of mathematics. Elements of the Theory of Numbers is a superb summary of classical material as well as

allowing the reader to take a look at the exciting role of analysis and algebra in number theory. * In-depth coverage of classical number theory * Thorough discussion of the theory of groups and rings * Includes application of

Taylor polynomials * Contains more advanced material than other texts * Illustrates the results of a theorem with an example * Excellent presentation of the standard computational exercises * Nearly 1000 problems--many

are proof-oriented, several others require the writing of computer programs to complete the computations * Clear and well-motivated presentation * Provides historical references noting distinguished number theory

luminaries such as Euclid, de Fermat, Hilbert, Brun, and Lehmer, to name a few * Annotated bibliographies appear at the end of all of the chapters This textbook covers a wide array of topics in analytic and

multiplicative number theory, suitable for graduate level courses. Extensively revised and extended, this Advanced Edition takes a deeper dive into the subject, with the elementary topics of the previous edition

making way for a fuller treatment of more advanced topics. The core themes of the distribution of prime numbers, arithmetic functions, lattice points, exponential sums and number fields now contain many more

details and additional topics. In addition to covering a range of classical and standard results, some recent work on a variety of topics is discussed in the book, including arithmetic functions of several variables, bounded gaps

between prime numbers à la Yitang Zhang, Mordell's method for exponential sums over finite fields, the resonance method for the Riemann zeta function, the Hooley divisor function, and many others. Throughout the book, the

emphasis is on explicit results. Assuming only familiarity with elementary number theory and analysis at an undergraduate level, this textbook provides an accessible gateway to a rich and active area of number theory.

With an abundance of new topics and 50% more exercises, all with solutions, it is now an even better guide for independent study.

Classical Theory

Analytic Number Theory

A Comprehensive Course in

Number Theory

Classical Theory of Arithmetic Functions

Analytic Number Theory distinguishes itself by the variety of tools it uses to establish results. One of the primary

attractions of this theory is its vast diversity of concepts and methods. The main goals of this book are to show the scope of the theory, both in classical and modern directions, and to exhibit its wealth and prospects,

beautiful theorems, and powerful techniques. The book is written with graduate students in mind, and the authors nicely balance clarity, completeness, and generality. The exercises in each section serve dual purposes,

some intended to improve readers' understanding of the subject and others providing additional information. Formal prerequisites for the major part of the book do not go beyond calculus, complex analysis,

integration, and Fourier series and integrals. In later chapters automorphic forms become important, with much of the necessary information about them included in two survey chapters.

Prime numbers are the multiplicative building blocks of natural numbers. Understanding their overall influence and especially their distribution gives rise to central questions in mathematics and physics. In

particular, their finer distribution is closely connected with the Riemann hypothesis, the most important unsolved problem in the mathematical world. This book comprehensively covers all the topics met in first courses on

multiplicative number theory and the distribution of prime numbers. The text is based on courses taught successfully over many years at the University of Michigan, Imperial College, London and Pennsylvania State

University.
Originally published in 1934, this volume presents the theory of the distribution of the prime numbers in the series of natural numbers. Despite being long out of print, it remains unsurpassed

as an introduction to the field. Number theory was once famously labeled the queen of mathematics by Gauss. The multiplicative structure of the integers in particular deals with many fascinating problems some

of which are easy to understand but very difficult to solve. In the past, a variety of very different techniques has been applied to further its understanding. Classical methods in analytic theory such as Mertens' theorem

and Chebyshev's inequalities and the celebrated Prime Number Theorem give estimates for the distribution of prime numbers. Later on, multiplicative structure of integers leads to multiplicative arithmetical

functions for which there are many important examples in number theory. Their theory involves the Dirichlet convolution product which arises with the inclusion of several summation techniques and a survey of

classical results such as Hall and Tenenbaum’s theorem and the M ö bius Inversion Formula. Another topic is the counting integer points close to smooth curves and its relation to the distribution of squarefree

numbers, which is rarely covered in existing texts. Final chapters focus on exponential sums and algebraic number fields. A number of exercises at varying levels are also included. Topics in Multiplicative Number Theory

introduces offers a comprehensive introduction into these topics with an emphasis on analytic number theory. Since it requires very little technical expertise it will appeal to a wide target group including upper

level undergraduates, doctoral and masters level students. Multiplicative Number Theory I Multiplicative Number Theory From Pythagoras to Riemann Quadratic Irrationals

Natural numbers are the

oldest human invention. This book describes their nature, laws, history and current status. It has seven chapters. The first five chapters contain not only the basics of elementary

number theory for the convenience of teaching and continuity of reading, but also many latest research results. The first time in history, the traditional name of the Chinese Remainder

Theorem is replaced with the Qin Jiushao Theorem in the book to give him a full credit for his establishment of this famous theorem in number theory. Chapter 6 is about the fascinating

congruence modulo an integer power, and Chapter 7 introduces a new problem extracted by the author from the classical problems of number theory, which is out of the combination of

additive number theory and multiplicative number theory.One feature of the book is the supplementary material after each section, there by broadening the reader's knowledge and

imagination. These contents either discuss the rudiments of some aspects or introduce new problems or conjectures and their extensions, such as perfect number problem, Egyptian fraction problem,

Goldbach's conjecture, the twin prime conjecture, the 3x + 1 problem, Hilbert Waring problem, Euler's conjecture, Fermat's Last Theorem, Laudau's problem and etc.This book is written

for anyone who loves natural numbers, and it can also be read by mathematics majors, graduate students, and researchers. The book contains many illustrations and tables. Readers can

appreciate the author's sensitivity of history, broad range of knowledge, and elegant writing style, while benefiting from the classical works and great achievements of masters in

number theory.
A new edition of a classical treatment of elliptic and modular functions with some of their number-theoretic applications, this text offers an updated bibliography and

an alternative treatment of the transformation formula for the Dedekind eta function. It covers many topics, such as Hecke's theory of entire forms with multiplicative Fourier

coefficients, and the last chapter recounts Bohr's theory of equivalence of general Dirichlet series. The exposition of the classical theory of algebraic numbers is clear and

thorough, and there is a large number of exercises as well as worked out numerical examples. A careful study of this book will provide a solid background to the learning

of more recent topics.
Comprehensive account of recent developments in arithmetic theory of modular forms, for graduates and researchers.
A Primer of Analytic Number

Theory
Topics in Classical Number Theory
Modular Functions and Dirichlet Series in Number Theory
Excursions in Multiplicative

Number Theory

This volume contains a collection of research and survey papers written by some of the most eminent mathematicians in the international community and is dedicated to Helmut Maier, whose own research has been

groundbreaking and deeply influential to the field. Specific emphasis is given to topics regarding exponential and trigonometric sums and their behavior in short intervals, anatomy of integers and cyclotomic polynomials, small

gaps in sequences of sifted prime numbers, oscillation theorems for primes in arithmetic progressions, inequalities related to the distribution of primes in short intervals, the Möbius function, Euler's totient function, the Riemann zeta function and the

Riemann Hypothesis. Graduate students, research mathematicians, as well as computer scientists and engineers who are interested in pure and interdisciplinary research, will find this volume a useful resource. Contributors to

this volume: Bill Allombert, Levent Alpoge, Nadine Amersi, Yuri Bilu, Régis de la Bretèche, Christian Elsholtz, John B. Friedlander, Kevin Ford, Daniel A. Goldston, Steven M. Gonek, Andrew Granville, Adam J. Harper, Glyn Harman, D. R. Heath-Brown,

Aleksandar Ivić, Geoffrey Iyer, Jerzy Kaczorowski, Daniel M. Kane, Sergei Konyagin, Dimitris Koukoulopoulos, Michel L. Lapidus, Oleg Lazarev, Andrew H. Ledoan, Robert J. Lemke Oliver, Florian Luca, James Maynard, Steven J. Miller, Hugh L.

Montgomery, Melvyn B. Nathanson, Ashkan Nikeghbali, Alberto Perelli, Amalia Pizarro-Madariaga, János Pintz, Paul Pollack, Carl Pomerance, Michael Th. Rassias, Maksym Radziwiłł, Joël Rivat, András Sárközy, Jeffrey Shallit, Terence Tao, Gérald

Tenenbaum, László Tóth, Tamar Ziegler, Liyang Zhang.

This basic introduction to number theory is ideal for those with no previous knowledge of the subject. The main topics of divisibility, congruences, and the distribution of prime numbers are

covered. Of particular interest is the inclusion of a proof for one of the most famous results in mathematics, the prime number theorem. With many examples and exercises, and only requiring knowledge of a little calculus and algebra, this book will suit

individuals with imagination and interest in following a mathematical argument to its conclusion.

This textbook offers a unique exploration of analytic number theory that is focused on explicit and realistic numerical bounds.

By giving precise proofs in simplified settings, the author strategically builds practical tools and insights for exploring the behavior of arithmetical functions. An active learning style is encouraged across nearly three hundred exercises, making this

an indispensable resource for both students and instructors. Designed to allow readers several different pathways to progress from basic notions to active areas of research, the book begins with a study of arithmetic functions and notions of arithmetical

interest. From here, several guided "walks" invite readers to continue, offering explorations along three broad themes: the convolution method, the Levin-Faĭnleĭb theorem, and the Mellin transform. Having followed any one of the walks, readers will

arrive at "higher ground" where they will find opportunities for extensions and applications, such as the Selberg formula, Exponential sums with arithmetical coefficients, and the Large Sieve Inequality. Methodology is emphasized

throughout, with frequent opportunities to explore numerically using computer algebra packages Pari/GP and Sage. Excursions in Multiplicative Number Theory is ideal for graduate students and upper-level undergraduate students who

are familiar with the fundamentals of analytic number theory. It will also appeal to researchers in mathematics and engineering interested in experimental techniques in this active area.

Quadratic Irrationals: An

Introduction to Classical Number Theory gives a unified treatment of the classical theory of quadratic irrationals. Presenting the material in a modern and elementary algebraic setting, the author focuses on equivalence, continued fractions, quadratic

characters, quadratic orders, binary quadratic forms, and class groups.T
Number Theory and Geometry: An Introduction to Arithmetic Geometry
The Distribution of Prime Numbers

Classical Theory of Algebraic Numbers

An Introductory Course in Elementary Number Theory

This valuable book focuses on a collection of powerful methods of analysis that yield deep number-

theoretical estimates. Particular attention is given to counting functions of prime numbers and multiplicative arithmetic functions. Both real variable (?elementary?) and complex variable (?analytic?) methods are employed. The reader is

assumed to have knowledge of elementary number theory (abstract algebra will also do) and real and complex analysis. Specialized analytic techniques, including transform and Tauberian methods, are developed as needed.Comments

and corrigenda for the book are found at http: //www.math.uiuc.edu/diamond/

"Number Theory Arising from Finite Fields: Analytic and Probabilistic Theory" offers a discussion of the advances and

developments in the field of number theory arising from finite fields. It emphasizes mean-value theorems of multiplicative functions, the theory of additive formulations, and the normal distribution of values from additive functions

Number Theory is a newly translated and revised edition of the most popular introductory textbook on the subject in Hungary. The book covers the usual topics of introductory number theory: divisibility, primes, Diophantine

equations, arithmetic functions, and so on. It also introduces several more advanced topics including congruences of higher degree, algebraic number theory, combinatorial number theory, primality testing, and cryptography.

The development is carefully laid out with ample illustrative examples and a treasure trove of beautiful and challenging problems. The exposition is both clear and precise. The book is suitable for both graduate and undergraduate courses

with enough material to fill two or more semesters and could be used as a source for independent study and capstone projects. Freud and Gyarmati are well-known mathematicians and mathematical educators in Hungary, and the

Hungarian version of this book is legendary there. The authors' personal pedagogical style as a facet of the rich Hungarian tradition shines clearly through. It will inspire and exhilarate readers.

An undergraduate-level 2003

introduction whose only prerequisite is a standard calculus course.

Introduction to Analytic and Probabilistic Number Theory

Opera de Cribro

1001 Problems in Classical Number Theory

An Introductory Course

[Hilbert's] style has not the terseness of many of our modem authors in mathematics, which is based on the assumption that printer's labor and paper are

costly but the reader's effort and time are not. H. Weyl [143] The purpose of this book is to describe the classical problems in additive number theory and to introduce the circle

method and the sieve method, which are the basic analytical and combinatorial tools used to attack these problems. This book is intended for students who want to lel?lll additive

number theory, not for experts who already know it. For this reason, proofs include many "unnecessary" and "obvious" steps; this is by design. The archetypical theorem in additive number

theory is due to Lagrange: Every nonnegative integer is the sum of four squares. In general, the set A of nonnegative integers is called an additive basis of order h if every nonnegative

integer can be written as the sum of h not necessarily distinct elements of A. Lagrange 's theorem is the statement that the squares are a basis of order four. The set A is called a basis

offinite order if A is a basis of order h for some positive integer h. Additive number theory is in large part the study of bases of finite order. The classical bases are the squares, cubes, and

higher powers; the polygonal numbers; and the prime numbers. The classical questions associated with these bases are Waring's problem and the Goldbach conjecture.

This volume focuses on the classical theory of number-theoretic functions emphasizing algebraic and multiplicative techniques. It contains many structure theorems basic to the study

of arithmetic functions, including several previously unpublished proofs. The author is head of the Dept. of Mathemati

Although it was in print for a short time only, the original

edition of Multiplicative Number Theory had a major impact on research and on young mathematicians. By giving a connected account of the large sieve and Bombieri's theorem,

Professor Davenport made accessible an important body of new discoveries. With this stimula tion, such great progress was made that our current understanding of these

topics extends well beyond what was known in 1966. As the main results can now be proved much more easily. I made the radical decision to rewrite §§23-29 completely for the second edition. In

making these alterations I have tried to preserve the tone and spirit of the original. Rather than derive Bombieri's theorem from a zero density estimate tor L timctions, as Davenport did,

I have chosen to present Vaughan'S elementary proof of Bombieri's theorem. This approach depends on Vaughan's simplified version of Vinogradov's method for estimating sums over prime

numbers (see §24). Vinogradov devised his method in order to estimate the sum LPH e(prx); to maintain the historical perspective I have inserted (in §§25, 26) a discussion of

this exponential sum and its application to sums of primes, before turning to the large sieve and Bombieri's theorem. Before Professor Davenport's untimely death in 1969,

several mathematicians had suggested small improvements which might be made in Multiplicative Number Theory, should it ever be reprinted. Multiplicative Number

Theory IClassical TheoryCambridge University Press
Advanced Edition
In Honor of Helmut Maier's 60th Birthday
An Introduction to

Mathematics

Introduction to Siegel Modular Forms and Dirichlet Series

This book is an introduction to analytic number theory suitable

for beginning graduate students. It covers everything one expects in a first course in this field, such as growth of arithmetic functions, existence of

primes in arithmetic progressions, and the Prime Number Theorem. But it also covers more challenging topics that might be used in a second course, such as

the Siegel-Walfisz theorem, functional equations of L-functions, and the explicit formula of von Mangoldt. For students with an interest in

Diophantine analysis, there is a chapter on the Circle Method and Waring's Problem. Those with an interest in algebraic number theory may find the chapter on

the analytic theory of number fields of interest, with proofs of the Dirichlet unit theorem, the analytic class number formula, the functional equation

of the Dedekind zeta function, and the Prime Ideal Theorem. The exposition is both clear and precise, reflecting careful attention to the needs of the reader. The

text includes extensive historical notes, which occur at the ends of the chapters. The exercises range from introductory problems and standard problems in analytic

number theory to interesting original problems that will challenge the reader. The author has made an effort to provide clear explanations for the

techniques of analysis used. No background in analysis beyond rigorous calculus and a first course in complex function theory is assumed.

This book is a revised and greatly expanded version of our book Elements of Number Theory published in 1972. As with the first book the primary

audience we envisage consists of upper level undergraduate mathematics majors and graduate students. We have assumed some familiarity with the

material in a standard undergraduate course in abstract algebra. A large portion of Chapters 1-11 can be read even without such background with the aid

of a small amount of supplementary reading. The later chapters assume some knowledge of Galois theory, and in Chapters 16 and 18 an acquaintance with the

theory of complex variables is necessary. Number theory is an ancient subject and its content is vast. Any intro ductory book must, of necessity, make a

very limited selection from the fascinat ing array of possible topics. Our focus is on topics which point in the direction of algebraic number theory

and arithmetic algebraic geometry. By a careful selection of subject matter we have found it possible to exposit some rather advanced material without requiring very

much in the way oftechnical background. Most of this material is classical in the sense that is was dis covered during the nineteenth century and earlier, but

it is also modern because it is intimately related to important research going on at the present time.

The new edition of this thorough examination of

the distribution of prime numbers in arithmetic progressions offers many revisions and corrections as well as a new section recounting recent works

in the field. The book covers many classical results, including the Dirichlet theorem on the existence of prime numbers in arithmetical progressions and the

theorem of Siegel. It also presents a simplified, improved version of the large sieve method. Number Theory is more than a comprehensive

treatment of the subject. It is an introduction to topics in higher level mathematics, and unique in its scope; topics from analysis, modern

algebra, and discrete mathematics are all included. The book is divided into two parts. Part A covers key concepts of number theory and could serve

as a first course on the subject. Part B delves into more advanced topics and an exploration of related mathematics. The prerequisites for this

self-contained text are elements from linear algebra. Valuable references for the reader are collected at the end of each chapter. It is suitable as an

introduction to higher level mathematics for undergraduates, or for self-study.

Analytic And Probabilistic Theory

Arithmetic Tales

Modular Forms and Galois Cohomology

A Classical Introduction to Modern Number Theory

This is a true masterpiece that will prove to be indispensable to the serious researcher for many years

to come. --Enrico Bombieri, Institute for Advanced Study This is a truly comprehensive account of sieves and their applications, by two of the world's greatest authorities. Beginners will find a thorough introduction to the

subject, with plenty of helpful motivation. The more practised reader will appreciate the authors' insights into some of the more mysterious parts of the theory, as well as the wealth of new examples. --Roger Heath-Brown,

University of Oxford, Fellow of Royal Society This is a comprehensive and up-to-date treatment of sieve methods. The theory of the sieve is developed thoroughly with complete and accessible proofs of the basic

theorems. Included is a wide range of applications, both to traditional questions such as those concerning primes, and to areas previously unexplored by sieve methods, such as elliptic curves, points on cubic surfaces and

quantum ergodicity. New proofs are given also of some of the central theorems of analytic number theory; these proofs emphasize and take advantage of the applicability of sieve ideas. The book contains numerous

comments which provide the reader with insight into the workings of the subject, both as to what the sieve can do and what it cannot do. The authors reveal recent developements by which the parity barrier can be

breached, exposing golden nuggets of the subject, previously inaccessible. The variety in the topics covered and in the levels of difficulty encountered makes this a work of value to novices and experts alike, both as an

educational tool and a basic reference.

A 2006 text based on courses taught successfully over many years at Michigan, Imperial College and Pennsylvania State. These notes serve as course notes

for an undergraduate course in number theory. Most if not all universities worldwide offer introductory courses in number theory for math majors and in many cases as an elective course. The notes contain a useful

introduction to important topics that need to be addressed in a course in number theory. Proofs of basic theorems are presented in an interesting and comprehensive way that can be read and understood even by non-majors

with the exception in the last three chapters where a background in analysis, measure theory and abstract algebra is required. The exercises are carefully chosen to broaden the understanding of the concepts. Moreover, these notes

shed light on analytic number theory, a subject that is rarely seen or approached by undergraduate students. One of the unique characteristics of these notes is the careful choice of topics and its importance in the theory of

numbers. The freedom is given in the last two chapters because of the advanced nature of the topics that are presented.

This is a self-contained introduction to analytic methods in number theory, assuming on

the part of the reader only what is typically learned in a standard undergraduate degree course. It offers to students and those beginning research a systematic and consistent account of the subject but will also be a

convenient resource and reference for more experienced mathematicians. These aspects are aided by the inclusion at the end of each chapter a section of bibliographic notes and detailed exercises.

Number Theory Arising From Finite Fields

An Introduction to Classical Number Theory

A Course in Analytic Number Theory

Elements of the Theory of

Numbers

"This book is the first volume of a two-volume textbook for undergraduates and is indeed the crystallization of a course offered by the author at the California Institute of Technology to undergraduates without any previous knowledge of number theory. For this

reason, the book starts with the most elementary properties of the natural integers. Nevertheless, the text succeeds in presenting an enormous amount of material in little more than 300 pages."-—MATHEMATICAL REVIEWS

Geometry and the theory of numbers

are as old as some of the oldest historical records of humanity. Ever since antiquity, mathematicians have discovered many beautiful interactions between the two subjects and recorded them in such classical texts as Euclid's Elements and Diophantus's Arithmetica. Nowadays,

the field of mathematics that studies the interactions between number theory and algebraic geometry is known as arithmetic geometry. This book is an introduction to number theory and arithmetic geometry, and the goal of the text is to use geometry as the motivation to prove

the main theorems in the book. For example, the fundamental theorem of arithmetic is a consequence of the tools we develop in order to find all the integral points on a line in the plane. Similarly, Gauss's law of quadratic reciprocity and the theory of continued fractions naturally arise

when we attempt to determine the integral points on a curve in the plane given by a quadratic polynomial equation. After an introduction to the theory of diophantine equations, the rest of the book is structured in three acts that correspond to the study of the integral and rational solutions of

linear, quadratic, and cubic curves, respectively. This book describes many applications including modern applications in cryptography; it also presents some recent results in arithmetic geometry. With many exercises, this book can be used as a text for a first course in number

theory or for a subsequent course on arithmetic (or diophantine) geometry at the junior-senior level.

A Modern Introduction To Classical Number Theory

Topics in Multiplicative Number Theory

Additive Number Theory The Classical Bases