Topos Theory

This text introduces topos theory, a development in category theory that unites important but seemingly diverse notions from algebraic geometry, set theory, and intuitionistic logic. Topics include local set theories, fundamental properties of toposes, sheaves, local-valued sets, and natural and real numbers in local set theories. 1988 edition.

In the last five decades various attempts to formulate theories of quantum gravity have been made, but none has fully succeeded in becoming the quantum theory of gravity. One possible explanation for this failure might be the unresolved fundamental issues in quantum theory as it stands now. Indeed, most approaches to quantum gravity adopt standard quantum theory as their starting point, with the hope that the theory ' s unresolved issues will get solved along the way. However, these fundamental issues may need to be solved before attempting to define a quantum theory of gravity. The present text adopts this point of view, addressing the following basic questions: What are the main conceptual issues in quantum theory? How can these issues be solved within a new theoretical framework of quantum theory? A possible way to overcome critical issues in present-day quantum physics – such as a priori assumptions about space and time that are not compatible with a theory of quantum gravity, and the impossibility of talking about systems without reference to an external observer – is through a reformulation of quantum theory in terms of a different mathematical framework called topos theory. This course-tested primer sets out to explain to graduate students and newcomers to the field alike, the reasons for choosing topos theory to resolve the above-mentioned issues and how it brings quantum physics back to looking more like a " neo-realist " classical physics theory again.

Category theory is a general mathematical theory of structures and of structures of structures. It occupied a central position in contemporary mathematics as well as computer science. This book describes the history of category theory whereby illuminating its symbiotic relationship to algebraic topology, homological algebra, algebraic geometry and mathematical logic and elaboratively develops the connections with the epistemological significance.

Mathematicians interested in understanding the directions of current research in set theory will not want to overlook this book, which contains the proceedings of the AMS Summer Research Conference on Axiomatic Set Theory, held in Boulder, Colorado, June 19-25, 1983. This was the first large meeting devoted exclusively to set theory since the legendary 1967 UCLA meeting, and a large majority of the most active research mathematicians in the field participated. All areas of set theory, including constructibility, forcing, combinatorics and descriptive set theory, were represented; many of the papers in the proceedings explore connections between areas. Readers should have a background of graduate-level set theory. There is a paper by S. Shelah applying proper forcing to obtain consistency results on combinatorial cardinal 'invariants' below the continuum, and papers by R. David and S. Freidman on properties of $0^\No. $.Papers by A. Blass, H.D. Donder, T. Jech and W. Mitchell involve inner models with measurable cardinals and various combinatorial properties. T. Carlson largely solves the pin-up problem, and D. Velleman presents a novel construction of a Souslin tree from a morass. S. Todorcevic obtains the strong failure of the \qedprinciple from the Proper Forcing Axiom and A. Miller discusses properties of a new species of perfect-set forcing. H. Becker and A. Kechris attack the third Victoria Delfino problem while W. Zwicker looks at combinatorics on $P_\kappa(\lambda)$ and J. Henle studies infinite-exponent partition relations. A. Blass shows that if every vector space has a basis then AC holds. I. Anellis treats the history of set theory, and W. Fleissner presents set-theoretical axioms of use in general topology.

A First Course in Topos Quantum Theory
Seven Sketches in Compositionality
Category Theory in Context
Basic Category Theory
The Topos of Music

In 'Higher Topos Theory', Jacob Lurie presents the foundations of this theory using the language of weak Kan complexes introduced by Boardman and Vogt, and shows how existing theorems in algebraic topology can be reformulated and generalized in the theory's new language.

Introduction to concepts of category theory — categories, functors, natural transformations, the Yoneda lemma, limits and colimits, adjunctions, monads — revisits a broad range of mathematical examples from the categorical perspective. 2016 edition.

As its title suggests, this book is an introduction to three ideas and the connections between them. Before describing the content of the book in detail, we describe each concept briefly. More extensive introductory descriptions of each concept are in the introductions and notes to Chapters 2, 3 and 4. A topos is a special kind of category defined by axioms saying roughly that certain constructions one can make with sets can be done in the category. In that sense, a topos is a generalized set theory. However, it originated with Grothendieck and Giraud as an abstraction of the of the category of sheaves of sets on a topological space. Later, properties Lawvere and Tierney introduced a more general id~a which they called "elementary topos" (because their axioms did not quantify over sets), and they and other mathematicians developed the idea that a theory in the sense of mathematical logic can be regarded as a topos, perhaps after a process of completion. The concept of triple originated (under the name "standard construc in Godement's book on sheaf theory for the purpose of computing tions") sheaf cohomology. Then Peter Huber discovered that triples capture much of the information of adjoint pairs. Later Linton discovered that triples gave an equivalent approach to Lawverc's theory of equational theories (or rather the infinite generalizations of that theory). Finally, triples have turned out to be a very important tool for deriving various properties of toposes.

With contributions by numerous experts
Toposes and Local Set Theories
Category Theory
Foundations of Quantum Theory
Theories, Sites, Toposes

Sheaves in Geometry and Logic

The book covers elementary aspects of category theory and topos theory. It has few mathematical prerequisites, and uses categorical methods throughout rather than beginning with set theoretic foundations. It works with key notions such as cartesian closedness, adjunctions, regular categories, and the internal logic of a topos. Full statements and elementary proofs are given for the central theorems, including the fundamental theorem of toposes, the sheafification theorem, and the construction of Grothendieck toposes over any topos as base. Three chapters discuss applications of toposes in detail, namely to sets, to basic differential geometry, and to recursive analysis. - ;Introduction; PART I: CATEGORIES: Rudimentary structures in a category; Products, equalizers, and their duals; Groups; Sub-objects, pullbacks, and limits; Relations; Cartesian closed categories; Product operators and others; PART II: THE CATEGORY OF CATEGORIES: Functors and categories; Natural transformations; Adjunctions; Slice categories; Mathematical foundations; PART III: TOPOSES: Basics; The internal language; A soundness proof for topos logic; From the internal language to the topos; The fundamental theorem; External semantics; Natural number objects; Categories in a topos; Topologies; PART IV: SOME TOPOSES: Sets; Synthetic differential geometry; The effective topos; Relations in regular categories; Further reading; Bibliography; Index. -

Please note that the content of this book primarily consists of articles available from Wikipedia or other free sources online. Pages: 38. Chapters: Effective topos, Grothendieck topology, History of topos theory, Nisnevich topology, Subobject classifier.

This volume explores the many different meanings of the notion of the axiomatic method, offering an insightful historical and philosophical discussion about how these notions changed over the millennia. The author, a well-known philosopher and historian of mathematics, first examines Euclid, who is considered the father of the axiomatic method, before moving onto Hilbert and Lawvere. He then presents a deep textual analysis of each writer and describes how their ideas are different and even how their ideas progressed over time. Next, the book explores category theory and details how it has revolutionized the notion of the axiomatic method. It considers the question of identity/equality in mathematics as well as examines the received theories of mathematical structuralism. In the end, Rodin presents a hypothetical New Axiomatic Method, which establishes closer relationships between mathematics and physics. Lawvere's axiomatization of topos theory and Voevodsky's axiomatization of higher homotopy theory exemplify a new way of axiomatic theory building, which goes beyond the classical Hilbert-style Axiomatic Method. The new notion of Axiomatic Method that

emerges in categorical logic opens new possibilities for using this method in physics and other natural sciences. This volume offers readers a coherent look at the past, present and anticipated future of the Axiomatic Method.

An introduction to the theory of toposes which begins with illustrative examples and goes on to explain the underlying ideas of topology and sheaf theory as well as the general theory of elementary toposes and geometric morphisms and their relation to logic.

Geometric Logic, Classification, Harmony, Counterpoint, Motives, Rhythm

Model Theory and Topoi

An Invitation to Applied Category Theory

Higher Topos Theory

Handbook of Categorical Algebra: Volume 1, Basic Category Theory

First of a 3-volume work giving a detailed account of what should be known by all working in, or using category theory. Volume 1 covers basic concepts.

Category theory has provided the foundations for many of the twentieth century's greatest advances in pure mathematics. This concise, original text for a one-semester course on the subject is derived from courses that author Emily Riehl taught at Harvard and Johns Hopkins Universities. The treatment introduces the essential concepts of category theory: categories, functors, natural transformations, the Yoneda lemma, limits and colimits, adjunctions, monads, and other topics. Suitable for advanced undergraduates and graduate students in mathematics, the text provides tools for understanding and attacking difficult problems in algebra, number theory, algebraic geometry, and algebraic topology. Drawing upon a broad range of mathematical examples from the categorical perspective, the author illustrates how the concepts and constructions of category theory arise from and illuminate more basic mathematical ideas. Prerequisites are limited to familiarity with some basic set theory and logic.

Category Theory has developed rapidly. This book aims to present those ideas and methods which can now be effectively used by Mathe maticians working in a variety of other fields of Mathematical research. This occurs at several levels. On the first level, categories provide a convenient conceptual language, based on the notions of category, functor, natural transformation, contravariance, and functor category. These notions are presented, with appropriate examples, in Chapters I and II. Next comes the fundamental idea of an adjoint pair of functors. This appears in many substantially equivalent forms: That of universal construction, that of direct and inverse limit, and that of pairs offunctors with a natural isomorphism between corresponding sets of arrows. All these forms, with their interrelations, are examined in Chapters III to V. The slogan is "Adjoint functors arise everywhere". Alternatively, the *fundamental notion of category theory is that of a monoid -a set with a binary operation of multiplication which is associative and which has a unit; a category itself can be regarded as a sort of general ized monoid. Chapters VI and VII explore this notion and its generaliza tions. Its close connection to pairs of adjoint functors illuminates the ideas of universal algebra and culminates in Beck's theorem characterizing categories of algebras; on the other hand, categories with a monoidal structure (given by a tensor product) lead inter alia to the study of more convenient categories of topological spaces.*

This innovative monograph explores a new mathematical formalism in higher-order temporal logic for proving properties about the behavior of systems. Developed by the authors, the goal of this novel approach is to explain what occurs when multiple, distinct system components interact by using a category-theoretic description of behavior types based on sheaves. The authors demonstrate how to analyze the behaviors of elements in continuous and discrete dynamical systems so that each can be translated and compared to one another. Their temporal logic is also flexible enough that it can serve as a framework for other logics that work with similar models. The book begins with a discussion of behavior types, interval domains, and translation invariance, which serves as the groundwork for temporal type theory. From there, the authors lay out the logical preliminaries they need for their temporal modalities and explain the soundness of those logical semantics. These results are then applied to hybrid dynamical systems, differential equations, and labeled transition systems. A case study involving aircraft separation within the National Airspace System is provided to illustrate temporal type theory in action. Researchers in computer science, logic, and mathematics interested in topos-theoretic and category-theory-friendly approaches to system behavior will find this monograph to be an important resource. It can also serve as a supplemental text for a specialized graduate topics course.

A History and Philosophy of Category Theory

Toposes, Triples and Theories

Category Theory in Physics, Mathematics, and Philosophy

A First Introduction to Topos Theory

A Topos-Theoretic Approach to Systems and Behavior

Sheaves arose in geometry as coefficients for cohomology and as descriptions of the functions appropriate to various kinds of manifolds. Sheaves also appear in logic as carriers for models of set theory. This text presents topos theory as it has developed from the study of sheaves. Beginning with several examples, it explains the underlying ideas of topology and sheaf theory as well as the general theory of elementary toposes and geometric morphisms and their relation to logic.

The contributions gathered here demonstrate how categorical ontology can provide a basis for linking three important basic sciences: mathematics, physics, and philosophy. Category theory is a new formal ontology that shifts the main focus from objects to *processes. The book approaches formal ontology in the original sense put forward by the philosopher Edmund Husserl, namely as a science that deals with entities that can be exemplified in all spheres and domains of reality. It is a dynamic, processual, and non-substantial ontology in which all entities can be treated as transformations, and in which objects are merely the sources and aims of these transformations. Thus, in a rather surprising way, when employed as a formal ontology, category theory can unite seemingly disparate disciplines in contemporary science and the humanities, such as physics, mathematics and philosophy, but also computer and complex systems science.*

According to Grothendieck, the notion of topos is "the bed or deep river where come to be married geometry and algebra, topology and arithmetic, mathematical logic and category theory, the world of the continuous and that of discontinuous or discrete structures". It is what he had "conceived of most broad to perceive with finesse, by the same language rich of geometric resonances, an "essence" which is common to situations most distant from each other, coming from one region or another of the vast universe of mathematical things". The aim of this book is to present a theory and a number of techniques which allow to give substance to Grothendieck's vision by building on the notion of classifying topos educed by categorical logicians. Mathematical theories (formalized within first-order logic) give rise to geometric objects called sites; the passage from sites to their associated toposes embodies the passage from the logical presentation of theories to their mathematical content, i.e. from syntax to semantics. The essential ambiguity given by the fact that any topos is associated in general with an infinite number of theories or different sites allows to study the relations between different theories, and hence the theories themselves, by using toposes as 'bridges' between these different presentations. The expression or calculation of invariants of toposes in terms of the theories associated with them or their sites of definition generates a great number of results and notions varying according to the different types of presentation, giving rise to a veritable mathematical morphogenesis.

Examining archival material and post-war scholarly and popular literature, Kranjc describes the often sharp divide between Communist-era interpretations of collaboration and those of their émigré anti-Communist opponents.

Axiomatic Method and Category Theory

Higher Topos Theory (AM-170)

Elementary Categories, Elementary Toposes

The Categorial Analysis of Logic

A Collection of Lectures by Variuos Authors

This is the first volume of the second edition of the now classic book “The Topos of Music”. The author explains the theory's conceptual framework of denotators and forms, the classification of local and global musical objects, the mathematical models of harmony and counterpoint, and topologies for rhythm and motives.

This book studies the foundations of quantum theory through its relationship to classical physics. This idea goes back to the Copenhagen Interpretation (in the original version due to Bohr and Heisenberg), which the author relates to the mathematical formalism of operator algebras originally created by von Neumann. The book therefore includes comprehensive appendices on functional analysis and C-algebras, as well as a briefer one on logic, category theory, and topos theory. Matters of foundational as well as mathematical interest that are covered in detail include symmetry (and its "spontaneous" breaking), the measurement problem, the Kochen-Specker, Free Will, and Bell Theorems, the Kadison-Singer conjecture, quantization, indistinguishable particles, the quantum theory of large systems, and quantum logic, the latter in connection with the topos approach to quantum theory. This book is Open Access under a CC BY licence.*

Part I indicates that typed-calculi are a formulation of higher-order logic, and cartesian closed categories are essentially the same. Part II demonstrates that another formulation of higher-order logic is closely related to topos theory.

Category Theory for the Sciences

Applications to Algebra, Logic and Topology. Proceedings of the International Conference Held at Gummersbach, July 6-10, 1981

Basic Category Theory for Computer Scientists

Tool and Object

What is Category Theory?

Focusing on topos theory's integration of geometric and logical ideas into the foundations of mathematics and theoretical computer science, this volume explores internal category theory, topologies and sheaves, geometric morphisms, and other subjects. 1977 edition.

An introduction to category theory as a rigorous, flexible, and coherent modeling language that can be used across the sciences. Category theory was invented in the 1940s to unify and synthesize different areas in mathematics, and it has proven remarkably successful in enabling powerful communication between disparate fields and subfields within mathematics. This book shows that category theory can be useful outside of mathematics as a rigorous, flexible, and coherent modeling language throughout the sciences. Information is inherently dynamic; the same ideas can be organized and reorganized in countless ways, and the ability to translate between such organizational structures is becoming increasingly important in the sciences. Category theory offers a unifying framework for information modeling that can facilitate the translation of knowledge between disciplines. Written in an engaging and straightforward style, and assuming little background in mathematics, the book is rigorous but accessible to non-mathematicians. Using databases as an entry to category theory, it begins with sets and functions, then introduces the reader to notions that are fundamental in mathematics: monoids, groups, orders, and graphs—categories in disguise. After explaining the “ big

three " concepts of category theory—categories, functors, and natural transformations—the book covers other topics, including limits, colimits, functor categories, sheaves, monads, and operads. The book explains category theory by examples and exercises rather than focusing on theorems and proofs. It includes more than 300 exercises, with solutions. Category Theory for the Sciences is intended to create a bridge between the vast array of mathematical concepts used by mathematicians and the models and frameworks of such scientific disciplines as computation, neuroscience, and physics.

Category theory reveals commonalities between structures of all sorts. This book shows its potential in science, engineering, and beyond.

Basic Category Theory for Computer Scientists provides a straightforward presentation of the basic constructions and terminology of category theory, including limits, functors, natural transformations, adjoints, and cartesian closed categories. Category theory is a branch of pure mathematics that is becoming an increasingly important tool in theoretical computer science, especially in programming language semantics, domain theory, and concurrency, where it is already a standard language of discourse. Assuming a minimum of mathematical preparation, Basic Category Theory for Computer Scientists provides a straightforward presentation of the basic constructions and terminology of category theory, including limits, functors, natural transformations, adjoints, and cartesian closed categories. Four case studies illustrate applications of category theory to programming language design, semantics, and the solution of recursive domain equations. A brief literature survey offers suggestions for further study in more advanced texts. Contents Tutorial · Applications · Further Reading

An Introduction

Proceedings of the International Conference held in Como, Italy, July 22-28, 1990

Volume 2

Geometric Logic of Concepts, Theory, and Performance

Mathematical Applications of Category Theory

Higher category theory is generally regarded as technical and forbidding, but part of it is considerably more tractable: the theory of infinity-categories, higher categories in which all higher morphisms are assumed to be invertible. In Higher Topos Theory, Jacob Lurie presents the foundations of this the using the language of weak Kan complexes introduced by Boardman and Vogt, and shows how existing theorems in algebraic topology can be reformu and generalized in the theory's new language. The result is a powerful theory with applications in many areas of mathematics. The book's first five cha give an exposition of the theory of infinity-categories that emphasizes their role as a generalization of ordinary categories. Many of the fundamental ide from classical category theory are generalized to the infinity-categorical setting, such as limits and colimits, adjoint functors, ind-objects and pro-object locally accessible and presentable categories, Grothendieck fibrations, presheaves, and Yoneda's lemma. A sixth chapter presents an infinity-categori version of the theory of Grothendieck topoi, introducing the notion of an infinity-topos, an infinity-category that resembles the infinity-category of topological spaces in the sense that it satisfies certain axioms that codify some of the basic principles of algebraic topology. A seventh and final chapte presents applications that illustrate connections between the theory of higher topoi and ideas from classical topology.

Topos Theory is an important branch of mathematical logic of interest to theoretical computer scientists, logicians and philosophers who study the foundations of mathematics, and to those working in differential geometry and continuum physics. This compendium contains material that was previou available only in specialist journals. This is likely to become the standard reference work for all those interested in the subject.

Higher Topos Theory (AM-170)Princeton University Press

With one exception, these papers are original and fully refereed research articles on various applications of Category Theory to Algebraic Topology, Lo and Computer Science. The exception is an outstanding and lengthy survey paper by Joyal/Street (80 pp) on a growing subject: it gives an account of classical Tannaka duality in such a way as to be accessible to the general mathematical reader, and to provide a key for entry to more recent developm and quantum groups. No expertise in either representation theory or category theory is assumed. Topics such as the Fourier cotransform, Tannaka du for homogeneous spaces, braided tensor categories, Yang-Baxter operators, Knot invariants and quantum groups are introduced and studies. From th Contents: P.J. Freyd: Algebraically complete categories.- J.M.E. Hyland: First steps in synthetic domain theory.- G. Janelidze, W. Tholen: How algebra is the change-of-base functor?.- A. Joyal, R. Street: An introduction to Tannaka duality and quantum groups.- A. Joyal, M. Tierney: Strong stacks andclassifying spaces.- A. Kock: Algebras for the partial map classifier monad.- F.W. Lawvere: Intrinsic co-Heyting boundaries and the Leibniz rule in certain toposes.- S.H. Schanuel: Negative sets have Euler characteristic and dimension.-

Topos Theory

Effective Topos, Grothendieck Topology, History of Topos Theory, Nisnevich Topology, Subobject Classifier

Introduction to Higher-Order Categorical Logic

From Classical Concepts to Operator Algebras

Temporal Type Theory

This advanced course, a sequel to the first volume of this lecture series on topos quantum theory, delves deeper into the theory, addressing further technical aspects and recent advances. These include, but are not limited to, the development of physical quantities and self-adjoint operators; insights into the quantization process; the description of an alternative, covariant version of topos quantum theory; and last but not least, the development of a new concept of spacetime. The book builds on the concepts introduced in the first volume (published as Lect. Notes Phys. 868), which presents the main building blocks of the theory and how it could provide solutions to interpretational problems in quantum theory, such as: What are the main conceptual issues in quantum theory? And how can these issues be solved within a new theoretical framework of quantum theory? These two volumes together provide a complete, basic course on topos quantum theory, offering a set of mathematical tools to readers interested in tackling fundamental issues in quantum theory in general, and in quantum gravity in particular. From the reviews of the first volume: The book is self-contained and can be used as a textbook or self-study manual teaching the usage of category theory and topos theory, in particular in theoretical physics or in investigating the foundations of quantum theory in mathematically rigorous terms. [The] book is a very welcome contribution. Frank Antonsen, Mathematical Reviews, December, 2013

The first of its kind, this book presents a widely accessible exposition of topos theory, aimed at the philosopher-logician as well as the mathematician. It is suitable for individual study or use in class at the graduate level (it includes 500 exercises). It begins with a fully motivated introduction to category theory itself, moving always from the particular example to the abstract concept. It then introduces the notion of elementary topos, with a wide range of examples and goes on to develop its theory in depth, and to elicit in detail its relationship to Kripke's intuitionistic semantics, models of classical set theory and the conceptual framework of sheaf theory (``localization'' of truth). Of particular interest is a Dedekind-cuts style construction of number systems in topoi, leading to a model of the intuitionistic continuum in which a ``Dedekind-real'' becomes represented as a ``continuously-variable classical real number''. The second edition contains a new chapter, entitled Logical Geometry, which introduces the reader to the theory of geometric morphisms of Grothendieck topoi, and its model-theoretic rendering by Makkai and Reyes. The aim of this chapter is to explain why Deligne's theorem about the existence of points of coherent topoi is equivalent to the classical Completeness theorem for ``geometric'' first-order formulae.

A short introduction ideal for students learning category theory for the first time.

Sketches of an Elephant: A Topos Theory Compendium

A Second Course in Topos Quantum Theory

Topoi

Relating and studying mathematical theories through topos-theoretic 'bridges'

Topos Theory, Why?